12B

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Tuesday, January 25, 1966 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 A diagonal of a rectangular parallelepiped is 11 and the dimensions of its base are 2 and 9. Find the length of the altitude of the parallelepiped.

2 From point P within dihedral angle $A-BC-D$, perpendicular lines PE and PF are drawn to the faces of the dihedral angle. If $\angle EPF = 130°$, find the number of degrees in dihedral angle $A-BC-D$.

3 The area of a great circle of a sphere is 40. Find the area of the sphere.

4 The dimensions of the base of a rectangular solid are 6 and 10. The total area of the solid is 280. Find the height of the solid.

5 The sides of a rectangle are 12 inches and 7 inches. When the rectangle is rotated through 360° about the 12-inch side as an axis, it forms solid A. When it is rotated through 360° about the 7-inch side as an axis, it forms solid B. Find the ratio of the volume of solid A to the volume of solid B.

6 The radius of a sphere is 24. How far from the center of the sphere must a plane be drawn to determine a small circle, the area of which will be one-half the area of the great circle? [Express answer in radical form.]

7 A pyramid of altitude 12 is cut by a plane parallel to the base at a point 2 inches from the vertex. If the area of the section is A, express the area of the base in terms of A.

8 In spherical triangle ABC, angle $A =$ angle $B = 90°$. The area of $\triangle ABC$ equals the area of a lune whose angle is 30°. Find the number of degrees in angle C.

9 The volume of a sphere is 36π. Express in terms of π the area of the sphere.

10 An equilateral spherical triangle whose angles are each 100° is drawn on a sphere whose radius is 6. Express in terms of π the area of the triangle.

11 The base of a pyramid is a right triangle whose legs are 5 and 9. The altitude of the pyramid is 16. Find the volume of the pyramid.

12 A right circular cylinder has a radius 2 and an altitude 5. Express in terms of π the total area of the cylinder.

13 An element of a right circular cone is two-thirds d, the diameter of the base. Express in terms of d the lateral area of the cone.

14 Find the length of an edge of a regular tetrahedron whose total area is $256\sqrt{3}$.

15 Line segment AB meets plane M at point B. The inclination of AB to M is 60° and the distance from A to M is 8. Find to the *nearest tenth* the length of AB.

16 A triangle with sides 5, 12 and 13 is revolved through 360° about the shortest side as an axis. Express in terms of π the volume of the solid generated.

17 Two sides of a spherical triangle are 110° and 130°, respectively. The third side is more than 20° and less than $x°$. Find the value of x.

18 The lateral area of a regular square pyramid is twice the area of the base. Find the number of degrees in the dihedral angle between the base and a lateral face.

Directions (19–27): Write on the separate answer sheet the *number* preceding the word or expression that best completes the statement or answers the question.

19 Two distinct lines are always parallel if they
1 are perpendicular to the same line
2 are parallel to the same plane
3 are parallel to the same line
4 lie one in each of two parallel planes

20 It is possible to have a spherical triangle whose sides are
(1) 90°, 120°, 150°   (3) 50°, 80°, 125°
(2) 60°, 60°, 130°   (4) 140°, 200°, 200°

21 One face angle of a trihedral angle is 110°. The other two face angles may be
(1) 140° and 100°   (3) 150° and 40°
(2) 140° and 130°   (4) 150° and 100°

22 Through a given external point, the number of lines that can be drawn parallel to both of two skew lines is
(1) 1   (3) 0
(2) 2   (4) an infinite number

23 A section of a cylinder made by a plane which contains an element is a quadrilateral. The quadrilateral is *always* a
1 square   3 rhombus
2 rectangle   4 parallelogram

24 A line l is perpendicular to plane M. The locus of points 7 inches from l and also 4 inches from M is
1 a straight line   3 a circle
2 two straight lines   4 two circles

25 Two planes are always parallel if they are
1 perpendicular to the same plane
2 perpendicular to the same line
3 tangent to the same sphere
4 parallel to the same line

26 The radius of the base of a zone of one base is 8 inches. The plane of the base is 6 inches from the center of the sphere. The number of square inches in the area of the zone is
(1) 60π (3) 80π
(2) 64π (4) 96π

27 A regular polyhedron with 12 vertices and 30 edges has
1 triangular faces   3 pentagonal faces
2 square faces   4 hexagonal faces

Directions (28–30): If the blank space in each statement below is replaced by the word *always, sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the separate answer sheet.

28 The intersection of a plane and a curved surface is . . . a curved line.

29 On a sphere, if two spherical triangles are each symmetric to a third spherical triangle, they are . . . congruent to each other.

30 An exterior angle of a spherical triangle is . . . equal to the sum of the opposite interior angles.

 [2]

Part II

Answer four questions from this part.   Show all work unless otherwise directed.

31 Prove *either* a or b but *not* both: [10]

   a Two lines perpendicular to the same plane are parallel.

OR

   b The sum of the angles of a spherical triangle is greater than 180° and less than 540°.

32 If a straight line and a plane are parallel, any plane perpendicular to the line is also perpendicular to the given plane.   [10]

33 Given two parallel planes m and n perpendicular to line l at points A and B, respectively; P is the midpoint of line segment AB.

   a Describe fully the locus of points in space equidistant from planes m and n.   [2]

   b Describe fully the locus of points in space at a distance d from line l.   [2]

   c Describe fully the locus of points in space at a distance r from point P.   [2]

   d [Write the number 1, 2, 3 or 4 on your answer paper after the letter d.]   [2]

   If the length of r is greater than the length of d, then the locus common to parts b and c is

   (1) one straight line   (3) one circle
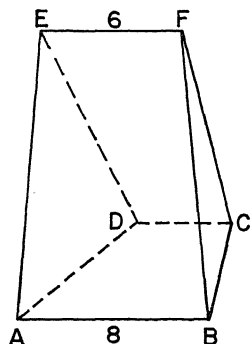   (2) two straight lines   (4) two circles

   e [Write the number 1, 2, 3 or 4 on your answer paper after the letter e.]   [2]

   If the length of d is equal to the length of r, then the locus common to parts a, b and c is

   (1) one circle   (3) one straight line
   (2) two circles   (4) two straight lines

34 In the prismatoid shown, ABCD is a trapezoid and line EF is parallel to the plane of ABCD.

The bases of the trapezoid are 4 and 8, respectively, and the altitude is 4. Line EF is 6 and is at a distance of 12 from ABCD.

Find the volume of the prismatoid.   [10]

35 The angles of a spherical triangle are in the ratio 2 : 3 : 3. The ratio of the area of the triangle to the area of the sphere on which it lies is 1 : 12. A zone equal in area to the triangle also lies on this sphere whose radius is 15.

   a Find the angles of the triangle.   [5]
   b Find the altitude of the zone.   [5]

36 A trough has the form of a right triangular prism, as shown. The rectangle at the top of the trough is l feet long and w feet wide, and the depth of the trough is h feet.
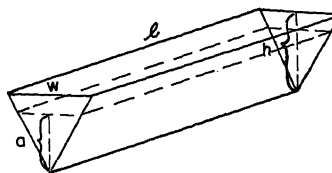
   a Show that the number of cubic feet of water, V, in the trough when it is filled to a depth of a feet is given by the formula $V = \dfrac{a^2 lw}{2h}$.   [7]

   b Find the number of gallons of water in the trough if l = 12 feet, w = 2 feet, h = 15 inches and a = 10 inches. [Use 7½ gallons = 1 cubic foot.]   [3]

*37 Answer *either* a or b but *not* both:

   a (1) Write an equation of the plane parallel to the yz-plane and passing through the point (4,—2,—4).   [2]

   (2) Write an equation of the plane parallel to the z-axis and passing through the points (3,0,0) and (0,1,0).   [2]

   (3) Write an equation of the plane passing through the points (5,0,0), (0,—1,0) and (0,0,—2).   [3]

   (4) Write an equation of the sphere with center at (0,0,0) and passing through the point (—4,8,—1).   [3]

OR

   b A right spherical triangle ABC has a = 40°, b = 51° and C = 90°. Find A and c, each to the nearest degree.   [5,5]

   * This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

12B

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 6

(2) 50

(3) 160

(4) 5

(5) $\frac{7}{12}$

(6) $12\sqrt{2}$ *or* $\sqrt{288}$

(7) $36A$

(8) 60

(9) 36π

(10) 24π

(11) 120

(12) 28π

(13) $\frac{\pi d^2}{3}$

(14) 16

(15) 9.2

(16) 240π

(17) 120

(18) 60

(19) 3

(20) 3

(21) 1

(22) 3

(23) 4

(24) 4

(25) 2

(26) 3

(27) 1

(28) sometimes

(29) always

(30) never

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) *a* A plane through P parallel to m and n [2]
    b A cylindrical surface with axis l and radius d [2]
    c A spherical surface with center P and radius r [2]
    d 4   [2]
    e 1   [2]

(34) 144   [10]

(35) *a* 60°, 90°, 90°   [5]
    b $2\frac{1}{2}$ [5]

(36) *b* 50   [3]

*(37) *a* (1) $x = 4$ [2]
     (2) $x + 3y = 3$ [2]
     (3) $2x - 10y - 5z = 10$ [3]
     (4) $x^2 + y^2 + z^2 = 81$ [3]
   b 47   [5]
     61   [5]