REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS 12B (Solid Geometry)

Thursday, January 24, 1963 — 1:15 to 4:15 p.m., only

Name of pupil	
Name and author of textbook used	
Name of teacher	
Part I	
Answer all questions in this part. Each correct answer will receive 2 credits. be allowed. Unless otherwise specified, answers may be left in terms of π or in respectively.	No partial credit wil adical form.
1 The dimensions of a rectangular parallelepiped are b , $2b$ and $3b$. Express the length of a diagonal of the parallelepiped in terms of b .	1
2 A lune with an area of 8π square inches is drawn on a sphere of radius 3 inches. Find the number of degrees in the angle of the lune.	2
3 Find to the nearest cubic foot the number of cubic feet of metal needed for 1,000 feet of solid metal rodding 1 inch in diameter. [Use the approximation $\pi = \frac{22}{7}$.]	3
4 Express the volume V of a cone of revolution in terms of the slant height l and the radius r of the base.	4
5 One of the regular polyhedrons has 12 edges and 6 vertices. How many faces does it have?	5
6 The projection of line segment RS of length a upon plane m is equal to $\frac{1}{2}a$. Find the number of degrees in the angle which the line determined by R and S makes with plane m .	6
7 A geometric figure consists of triangle ABC , segment RA perpendicular to the plane of triangle ABC , and segment RB . Name the line segment which is skew to RA .	
8 The total area of a right circular cylinder is 250π square feet and the altitude is 20 feet. Find the number of feet in the radius of a base of	7
9 Find to the <i>nearest cubic foot</i> the volume of a frustum of a square pyramid whose base edges are 20 feet and 10 feet, respectively, and whose altitude is 6 feet.	9

10	The area of a sphere of radius a is nume a sphere of radius b . Express a in term	erically equal to the volume of b .	10
11	Point P is located in the interior of a m and n . If P is 5 inches from m , $5 \vee 10^{-1}$ from the edge of the dihedral angle, fin	2 inches from n and 10 inches	47
	dihedral angle.		11
	In questions 12-14, P denotes 6 feet; the length of side s of its s altitude h', and the length of side s	a right prism which has an alt quare base is equal to 12 feet. of of its square base is equal to 1	Prism P' has an
12	If prism P' is equal in volume to prism	P, find h' to the nearest foot.	12
13	If prism P' is similar to prism P , find h	' to the nearest foot.	13
14	If a third prism Q is congruent to prism	n P , what is the total area of Q ?	14
15	Each base of a zone on a sphere has a of the sphere is 4 inches. Find the num of the zone.	radius of 2 inches. The radius aber of square inches in the area	15
16	The ratio of the volumes of two similar surface of the larger cylinder is 180, find	cylinders is 27:8. If the total is the total surface of the smaller.	16
17	The three sides of a spherical triangle a the number of spherical degrees in the	are 90°, 90° and 30°. What is area of the triangle?	17
18	A spherical plastic ball has a concentric If the diameter of the plastic ball is 10 i inches of plastic.	metal core of diameter 4 inches. inches, find the number of cubic	18
19	If an angle of a spherical triangle is 75° the side lying opposite it in its polar tri	, find the number of degrees in iangle.	19
	Directions (20-23): Indicate the country the line at the right the number 1, 2, 3 of		llowing by writing on
2	O If two face angles of a trihedral angle ar	e 135° and 100°, the third face	
	angle may be (1) 20°	(3) 125° (4) 170°	20
2	1 The locus of points which are equidistar and 4 units from a point on the intersect	nt from two intersecting planes tion of the planes consists of	
-	and 4 units from a point on the intersection (1) one circle (2) two circles	(3) two parallel lines (4) four parallel lines	21

	TWELFTH YEAR MATHEMATICS - 12B - continued	
22	The distance of point A from plane m is 4 inches. The locus of points at a distance 3 inches from point A and also 3 inches from plane m is a (1) point (2) straight line (3) circle (4) plane	22
	The base of a pyramid is a square. If each side of the base is doubled and the altitude of the pyramid is divided by two, the volume of the pyramid is	
	(1) multiplied by 1 (2) multiplied by 2 (3) multiplied by 4 (4) divided by 2	23
	A right triangle whose legs are 5 and 12 is rotated through 360° about the longer leg as an axis. What is the lateral area of the surface generated?	24
	A pyramid is 12 inches high, and the area of its base is 48 square inches. Find the number of square inches in the area of a section parallel to the base and 9 inches from it.	25
	On a sphere point A lies on a small circle whose poles are P and P' . If chord AP is 8 inches and chord AP' is 15 inches, find the number of inches in the length of the diameter of the sphere.	26
	Directions (27-30): If the blank space in each statement below is always, sometimes (but not always) or never, the resulting statement we word that will correctly complete each statement and write this word on the statement of the statement of the statement and write this word on the statement of the statement and write this word on the statement and write this word on the statement of the stat	ill be true. Select the
27	If, in a dihedral angle with faces M and N and edge q , line m in face M forms an angle of 60° with line n in face N and line m is perpendicular to line q , then the dihedral angle contains 60° .	27
28	Two pyramids which have congruent square bases and equal altitudes have equal lateral areas.	28
29	The locus of points equidistant from three given noncollinear points is a plane.	29
30	The intersection of three planes is a single point.	30



Part II

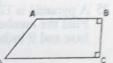
Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b: [10]

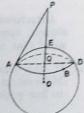
a If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane.

OR

- b The locus of points equally distant from two given points is the plane perpendicular to the line segment joining them at its midpoint.
- 32 Points A, B and C are midpoints of the three edges of a cube which meet at vertex P of the cube. The length of an edge of the cube is represented by e. Find in terms of e the a volume of the tetrahedron P-ABC b total surface area of the tetrahedron P-ABC
- 33 Prove that a section of an oblique triangular prism made by a plane parallel to one of its lateral edges is a parallelogram.
- 34 In the accompanying figure, ABCD is a trapezoid with bases AB and DC in the ratio 5:8, and angles B and C are right angles. Let the longer base be represented by 8x and the altitude by y.



- a Write in terms of x and y a formula for the volume of the solid formed by rotating the figure through 360° about the longer base as an axis. [4]
- b Using the formula obtained in answer to part a, find to the nearest integer the volume of the solid when x = 1.63 and y = 0.92. [Use the approximation $\pi = 3.14$.] [6]
- 35 In the accompanying diagram, P denotes the position of a light source which illuminates a zone on sphere O. The base of this zone is a circle ABD with center Q. Line PQO intersects the sphere at E. Let AO = r, PE = m, EQ = h and Z = the area of the zone. Show that $Z = \frac{2\pi r^2 m}{r + m}$



*36 Answer either a or b:

a Points A(2, 3, -4) and B(-1, 5, 6) are located with reference to three mutually perpendicular axes, x, y and z.

(1) Find the coordinates of the midpoint of line segment AB.

(2) Write an equation of the plane through point B parallel to the xy-plane.

(3) Determine the distance of point A from the plane described in part (2).

(4) Point A lies on a sphere with center at the origin. Write an equation of this

(5) With reference to the sphere described in part (4), point B lies (1) inside it, (2) on it, (3) outside it. [Write the number preceding the correct answer on your answer

paper after the number (5).]

b In spherical triangle ABC, angle $C = 90^{\circ}$, side $a = 118^{\circ}$ and angle $B = 12^{\circ}$.

(1) Find angle A to the nearest degree. [8] (2) Using the given data, write an equation that could be used to find side b.

* This question is based on optional topics in the syllabus.



FOR TEACHERS ONLY

12B

SCORING KEY

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Thursday, January 24, 1963-1:15 to 4:15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20-23, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1)
$$b\sqrt{14}$$

$$(2)$$
 80

$$(4) \, \, \tfrac{1}{3}\pi r^2 \, \sqrt{l^2 - r^2}$$

(10)
$$b\sqrt{\frac{b}{3}}$$
 or $\frac{b}{3}\sqrt{3b}$

(15)
$$32\pi\sqrt{3}$$

$$(18) 156\pi$$

$$(24) 65\pi$$



Part II

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(32)
$$a \frac{e^3}{48}$$
 [5]
 $b \frac{(3+\sqrt{3})e^2}{8}$ [5]

(36)
$$a$$
 (1) $(\frac{1}{2}, 4, 1)$ [2]
(2) $z = 6$ [2]
(3) 10 [2]
(4) $x^2 + y^2 + z^2 = 29$ [2]

(2) $\tan b = \sin a \tan B$ [2]