The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Wednesday, January 25, 1961 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A right section of a prism is a triangle whose sides are 5, 6 and 7. If a lateral edge of the prism is 4, find the lateral area of the prism. 1.....

2 The radius of the base of a right circular cone is 6 and its slant height is 10. Find the volume of the cone. 2.....

3 Find the lateral area of a frustum of a regular square pyramid whose base edges are 3 and 4, and whose slant height is 6. 3.....

4 The base of a right prism is a rectangle whose sides are 2 and 6. If a diagonal of the prism is 11, find the altitude of the prism. 4.....

5 A line 10 inches long is inclined to a plane at an angle of 44°. Find to the *nearest tenth* of an inch the length of the projection of the line on the plane. 5.....

6 A lune with an angle of 40° has an area of 16π square inches. Find the number of inches in the length of the radius of the sphere on which this lune is drawn. 6.....

7 The area of a sphere is 64π square inches. The altitude of a zone drawn on this sphere is 1 inch. Find the number of square inches in the area of the zone. 7.....

8 The sides of a spherical triangle are 75°, 85° and 100°. Find the number of spherical degrees in the area of its polar triangle. 8.....

9 The edge of a cube is 6. Find the volume of the sphere inscribed in the cube. 9.....

[OVER]

10 The polar distance of a small circle of a sphere is 30° and the radius of the sphere is 10. Find the length of the radius of the small circle. 10.....

11 The altitude of a pyramid is 10. Its base is a square, each of whose sides is 5. Find the area of the section made by a plane parallel to the base and at a distance of 6 from the vertex. 11.....

12 The slant height of a frustum of a cone of revolution is 10. If the radii of the bases are 3 and 5, find the lateral area of the frustum. 12.....

13 A solid spherical ball 2 inches in diameter weighs 10 pounds. Find the number of pounds in the weight of a ball of the same material which is 4 inches in diameter. 13.....

14 A rectangle whose sides are a and $2a$ is revolved through 360° about its longer side as an axis. Find the total area of the resulting solid. 14.....

15 Assuming the earth to be a sphere, what fractional part of the earth's surface is included between the meridians 10° west and 70° west? 15.....

16 The total area of a regular tetrahedron is $4\sqrt{3}$. Find an edge of the tetrahedron. 16.....

17 The slant height s of a right circular cone is equal to the diameter of the base. Express the lateral area of the cone in terms of s. 17.....

18 A dihedral angle of 120° is bisected by a plane. A point P in this plane is 8 inches from the edge of the angle. Find the number of inches in the distance from point P to either face of the dihedral angle. 18.....

19 A spherical triangle has angles of 60°, 80° and 100°. What fractional part of the area of the sphere is the area of this triangle? 19.....

Directions (20–25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

20 If a polyhedron is regular,
(1) it may have 10 faces
(2) its faces may be regular hexagons
(3) it may have 12 edges and 6 vertices
(4) it may have 8 edges and 8 vertices
20.....

21 The face angles of a trihedral angle may be
(1) 30°, 60°, 80°
(2) 10°, 80°, 100°
(3) 30°, 70°, 100°
(4) 100°, 120°, 140°
21.....

22 The locus of points equidistant from 2 intersecting planes and at a given distance from their line of intersection is
(1) 1 line
(2) 2 lines
(3) 3 lines
(4) 4 lines
22.....

23 Which statement is true for a spherical triangle?
(1) Its angles may be 30°, 60°, 90°.
(2) Its angles may be 110°, 120°, 130°.
(3) Its sides may be 30°, 60°, 90°.
(4) Its sides may be 110°, 120°, 130°.

23.....

24 If the radius of a circular cylinder is doubled and its altitude is divided by 2, the volume of the cylinder
(1) remains the same
(2) is divided by 2
(3) is doubled
(4) is multiplied by 4

24.....

25 A cone is circumscribed about a regular square pyramid whose base edge is s. The ratio of the volume of the cone to the volume of the pyramid is

(1) $\frac{\pi s^2}{2}$ (2) $\frac{\pi}{2}$ (3) $\frac{2}{\pi}$ (4) $\frac{\pi}{2s^2}$

25.....

Directions (26–30): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always), or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

26 If two planes are perpendicular to the same plane, they are ...perpendicular to each other. 26.....

27 If two lines are not in the same plane, a plane can ... be passed through one of these lines parallel to the other. 27.....

28 If two spherical triangles on equal spheres have the three sides of one equal to the three sides of the other, the triangles are ... congruent. 28.....

29 If two acute angles not in the same plane have their sides respectively parallel, they are ... equal. 29.....

30 A sphere can ... be circumscribed about a tetrahedron. 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either *a* or *b*: [10]

a Two planes perpendicular to the same line are parallel.

OR

b If a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle passing through these points.

32 $ABCD$ is a rectangle. PA is perpendicular to the plane of the rectangle and line PB is drawn. Prove that angle PBC is a right angle. [10]

33 Given plane P and line l parallel to P and 8 inches from P.

a Describe fully the locus of the centers of all spheres of radius r which are

(1) tangent to line l [2]

(2) tangent to plane P [2]

b Name the locus of points satisfying both conditions in part *a* if

(1) $r = 5$ inches [2]

(2) $r = 4$ inches [2]

c Give a value of r for which there will be no points satisfying both conditions in part *a*. [2]

34 A sphere whose diameter is 12 feet is illuminated by a point source of light 18 feet from the center of the sphere. Find the area of the portion of the sphere which is illuminated. [Answer may be left in terms of π.] [10]

35 One leg of an isosceles triangle is x, and it makes an angle of θ with the base. The triangle is revolved through 360° about the base as an axis.

a Show that the volume of the solid thus formed is given by the formula $V = \frac{2}{3}\pi x^3 \sin^2\theta \cos\theta$. [5]

b Find the volume to the *nearest integer* if $x = 3$ and $\theta = 58°$. [Use the approximation $\pi = 3.14$.] [5]

*36 Answer either *a* or *b*:

a Given spherical triangle ABC in which angle $C = 90°$, side $b = 110°$ and angle $A = 40°$.

(1) Find angle B to the *nearest degree*. [8]

(2) In spherical triangle ABC in which angle C is 90°, write an equation involving side b and angle A that could be used to find side c. [2]

OR

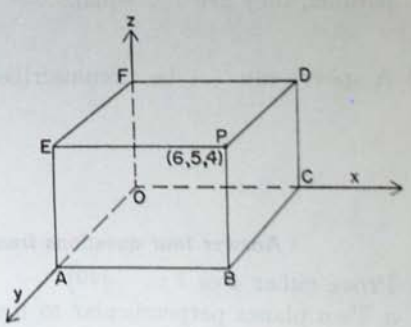
b The accompanying figure shows a rectangular parallelepiped whose base is in the xy-plane. Point P has the coordinates (6, 5, 4).

(1) Write the coordinates of the points A and D. [2]

(2) Find the length of diagonal AD. [3]

(3) Write an equation of the plane containing face $PBCD$. [2]

(4) Write an equation of the plane which passes through points A, F and C. [3]



*This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

12B

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Wednesday, January 25, 1961 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 72

(2) 96π

(3) 84

(4) 9

(5) 7.2

(6) 6

(7) 8π

(8) 100

(9) 36π

(10) 5

(11) 9

(12) 80π

(13) 80

(14) $6\pi a^2$

(15) $\frac{1}{6}$

(16) 2

(17) $\frac{\pi s^2}{2}$

(18) $4\sqrt{3}$

(19) $\frac{1}{12}$

(20) 3

(21) 1

(22) 4

(23) 2

(24) 3

(25) 2

(26) sometimes

(27) always

(28) sometimes

(29) always

(30) always

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

Part II

(33) *a* There are many ways of describing these loci. Each description should include shape and position. For instance, phrases such as the following should be allowed credit as indicated:

(1) a cylindrical surface with radius r and axis l [2]

(2) two planes parallel to P, one on either side, and at a distance r [2]

b (1) two parallel lines [2]

(2) one line [2]

c any value of r such that $0 < r < 4$ [2]

(34) 48π [10]

(35) *b* 22 [5]

(36) *a* (1) 103° [8]

(2) $\tan c = \dfrac{\tan b}{\cos A}$ *or* $\cot c = \dfrac{\cos A}{\tan b}$ [2]

b (1) A (0, 5, 0) and D (6, 0, 4) [2]

(2) $\sqrt{77}$ [3]

(3) $x = 6$ [2]

(4) $\dfrac{x}{6} + \dfrac{y}{5} + \dfrac{z}{4} = 1$ *or* $10x + 12y + 15z = 60$ [3]