The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Monday, January 25, 1960 — 1:15 to 4:15 p.m., only

Name of pupil ..... Name of school .....

Name and author of textbook used .....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A diagonal of a face of a cube is $\sqrt{2}$. Find a diagonal of the cube. 1.....

2 A right section of an oblique prism is a square whose edge is s and a lateral edge of the prism is $3s$. Express the lateral area in terms of s. 2.....

3 A line segment makes an angle of $77°$ with a plane and the length of its projection on the plane is 7.2. Find to the *nearest integer* the length of the segment. 3.....

4 The base of a pyramid is an equilateral triangle whose edge is 6, and the altitude of the pyramid is $2\sqrt{3}$. Find the volume of the pyramid. 4.....

5 The area of a sphere is six times the area of a zone which is drawn on its surface. Express the altitude of the zone in terms of the radius of the sphere. 5.....

6 Find the number of degrees in the sum of the angles of a spherical triangle whose area is one-tenth of the area of the sphere on which it is drawn. 6.....

7 A lune whose area is 2π square inches is drawn on a sphere of radius 6 inches. Find the number of degrees in an angle of the lune. 7.....

8 If one angle of a birectangular spherical triangle is $60°$, find the number of degrees in the longest side of its polar triangle. 8.....

9 The altitude of a cone of revolution is three times the radius (r) of its base. Express the volume of the cone in terms of r. 9.....

10 The altitude of a right circular cylinder is equal to the diameter of its base. The lateral area of the cylinder is 100π. Find the radius of the base. 10.....

11 The slant height of the frustum of a regular square pyramid is 10. An edge of the lower base is 6 and an edge of the upper base is 4. Find the lateral area of the frustum. 11.....

12 The altitude of a pyramid is 6 inches. The area of a section of this pyramid formed by a plane parallel to the base and 4 inches from the base is 2 square inches. Find the number of square inches in the base of the pyramid. 12.....

Directions (13–16) : If the blank space in each statement below is replaced by the word *always, sometimes* (but not always), or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

13 A plane is ... determined by two nonintersecting lines. 13.....

14 If two planes are perpendicular to each other, a line perpendicular to one of the planes is ... parallel to the other plane. 14.....

15 The projection of a rectangle on a plane oblique to the plane of the rectangle is ... a parallelogram. 15.....

16 If the altitude of a cone of revolution is equal to the radius of the base, the area of the base is ... equal to the lateral area of the cone. 16.....

Directions (17–20) : Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

17 Two regular prisms with bases of six and four sides, respectively, have equal altitudes and equal base edges. The ratio of their volumes is (1) $\sqrt{3}:4$ (2) $3:2$ (3) $3\sqrt{3}:2$ (4) $27:8$ 17.....

18 The number of faces in each of the three regular polyhedrons whose faces are equilateral triangles is (1) 4, 8 and 20 (2) 4, 6 and 8 (3) 4, 8 and 12 (4) 4, 12 and 20 18.....

19 Two face angles of a trihedral angle are 78° and 108°. The third face angle may be (1) 194° (2) 174° (3) 78° (4) 24° 19.....

20 The locus of points equally distant from two intersecting lines and also at a given distance d from their point of intersection is (1) one circle (2) two circles (3) two planes (4) four points 20.....

Part II

Answer five questions from this part. Show all work unless otherwise directed.

21 Prove: If a line is perpendicular to a plane, every plane passed through this line is perpendicular to the given plane. [10]

22 Prove: If the first of two spherical triangles is the polar triangle of the second, then the second is the polar of the first. [10]

23 The ratio of the altitude of a zone to the diameter of the sphere on which it is drawn is 1 : 5. The area of the zone is 80π.

a Find the area of the sphere. [5]

b If one of the bases of the zone is a great circle, find the area of the other base. [5]
[Answers may be expressed in terms of π.]

24 From a point within a dihedral angle, perpendicular lines are drawn to each face.

a Prove that the plane determined by these perpendiculars is perpendicular to the edge of the dihedral angle. [7]

b If the number of degrees in the dihedral angle is represented by n, express in terms of n the number of degrees in the angle formed by the two perpendicular lines. [3]

25 Lines s and t are perpendicular to plane M. The distance between the lines is 6. Each locus listed in column I is described briefly *once* and *only once* in column II. List the letters *a*–*e* on your answer paper, and after *each* letter write the *number* that indicates the description of that locus. [10]

Column I	*Column II*
a Locus of points at a distance of 4 from both s and t	(1) a line parallel to s
b Locus of points at a distance of 3 from both s and t	(2) two lines parallel to s
c Locus of points at a distance of 3 from M	(3) four lines parallel to s
d Locus of points at a distance of 3 from both s and M	(4) one point
e Locus of points at a distance of 3 from s, t and M	(5) two points
	(6) four points
	(7) two circles
	(8) a plane parallel to s
	(9) a plane parallel to M
	(10) two planes parallel to M

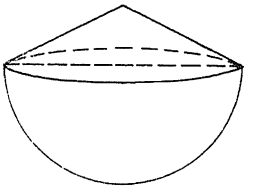
26 A regular pyramid has a pentagon for its base.

a Show that the area of the base is given by the formula $B = \frac{5}{4}e^2 \tan 54°$ where e = an edge of the base. [5]

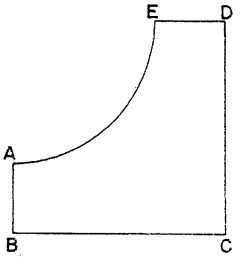
b If the slant height of the pyramid makes an angle of 54° with the altitude of the pyramid, show that the altitude is given by the formula $h = \frac{e}{2}$. [3]

c Using the formulas of parts *a* and *b*, write a formula for the volume of the pyramid in terms of the base edge e. [2]

27 A reservoir is to have the shape of a hemisphere of radius 21 feet surmounted by a right circular cone. The base of the cone coincides with the base of the hemisphere. Find the number of feet in the height of the cone in order that the reservoir may have a total volume of 23,100 cubic feet. [Use the approximation $\pi = \frac{22}{7}$.] [10]



28 In the figure at the right, B, C and D are right angles. $AB = ED = a$. AE is a quadrant of a circle whose radius is $2a$. Find in terms of a the total area of the solid formed by rotating the figure through 360° about AB as an axis. [Answer may be expressed in terms of π.] [10]



The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

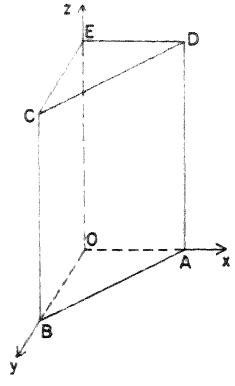
12B (Solid Geometry)

Monday, January 25, 1960 — 1:15 to 4:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in solid geometry by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in solid geometry. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one* or *both* may be substituted for *any one* or *two* of the questions on part II of the examination in solid geometry. Unless otherwise directed, answers may be left in radical form.

29 In the accompanying figure, a right prism has for its base an isosceles right triangle, one of whose equal sides is 3. The base of the prism is in the xy-plane with the vertex of the right angle at the origin. The altitude of the prism is 6.



a Write the coordinates of the points A and B. [2]

b Write an equation of the plane containing the face $ABCD$ of the prism. [3]

c Write an equation of the plane which contains the upper base CED of the prism. [2]

d Find the length of the diagonal BD. [3]

30 Given spherical triangle ABC in which angle $C = 90°$, angle $A = 104°$ and side $c = 132°$.

a Find side a to the *nearest degree*. [8]

b Using the given data, write an equation that could be used to find side b. [2]

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING

SG

SOLID GEOMETRY

and

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 17–20, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\sqrt{3}$

(2) $12s^2$

(3) 32

(4) 18

(5) $\frac{1}{3}r$

(6) 252

(7) 5

(8) 120

(9) πr^3

(10) 5

(11) 200

(12) 18

(13) sometimes

(14) sometimes

(15) always

(16) never

(17) 3

(18) 1

(19) 3

(20) 2

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

Part II

23 *a* 400π [5]
b 84π [5]

24 *b* $180 - n$ [3]

25 *a* 2 [2]
b 1 [2]
c 10 [2]
d 7 [2]
e 5 [2]

26 *c* $V = \frac{5}{24} e^3 \tan 54°$ [2]

27 8 [10]

28 $40\pi a^2$ [10]

Twelfth Year Mathematics (Solid Geometry)

29 *a* $A(3, 0, 0)$ and $B(0, 3, 0)$ [2]
b $x + y - 3 = 0$ [3]
c $z = 6$ [2]
d $\sqrt{54}$ *or* $3\sqrt{6}$ [3]

30 *a* 134 [8]
b $\tan b = \tan c \cos A$ [2]