The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# SOLID GEOMETRY

Thursday, January 22, 1959 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

## Part I

**Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.**

1 The area of the base of a pyramid is 24 and its altitude is 10. Find the volume of the pyramid. 1.....

2 Find the lateral area of a regular pentagonal prism whose lateral edge is 8 and whose base edge is 6. 2.....

3 The dimensions of the base of a rectangular solid are 9 and 12 and the diagonal of the solid is 17. Find the altitude. 3.....

4 A plane is passed parallel to the base of a pyramid and 3 inches from its vertex. The altitude of the pyramid is 5 inches and the area of its base is 50 square inches. Find the number of square inches in the area of the section cut by the parallel plane. 4.....

5 A base edge of a regular square pyramid is 6 and a lateral edge is 5. Find, to the *nearest degree,* the angle formed by the base and a lateral face. 5.....

6 Find the lateral area of the frustum of a regular triangular pyramid, if the base edges are 4 and 7, respectively, and the slant height is 8. 6.....

7 A right circular cylinder and a cone of revolution have equal bases and equal lateral areas. The altitude of the cylinder is 7. Find the slant height of the cone. 7.....

8 Find the volume of a sphere of radius 5. 8.....

9 The area of a lune is 30 spherical degrees. Find the number of degrees in each angle of an equilateral spherical triangle whose area equals the area of the lune. 9.....

10 The edges of the bases of a frustum of a regular square pyramid are 8 and 18, respectively. The slant height makes an angle of 60° with the base. Find the altitude of the frustum. 10.....

*Directions* (11–14): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

11 A diameter of a sphere is perpendicular to the planes of two great circles of the sphere. 11.....

12 If each of three lines is perpendicular to a fourth line at the same point, the three lines are coplanar. 12.....

13 If two lines are parallel to the same plane, they are parallel to each other. 13.....

14 If two spherical triangles on the same sphere have three angles of one equal respectively to three angles of the other, they are either congruent or symmetric. 14.....

*Directions* (15–20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

15 If line segment $AB$ is oblique to plane $m$, then the projection of $AB$ on $m$ is (*a*) a point (*b*) a line segment less than $AB$ (*c*) a line segment equal to $AB$ (*d*) a line segment greater than $AB$ 15.....

16 A cube is circumscribed about a sphere. Of the following ratios, the best approximation for the ratio of the surface of the sphere to the total surface of the cube is (*a*) 3:4 (*b*) 2:3 (*c*) 1:2 (*d*) 1:3 16.....

17 The total area of a regular icosahedron of edge 2 is (*a*) $8\sqrt{3}$ (*b*) $12\sqrt{3}$ (*c*) $15\sqrt{3}$ (*d*) $20\sqrt{3}$ 17.....

18 Two face angles of a trihedral angle are 100° and 130°. The third face angle may be (*a*) 140° (*b*) 130° (*c*) 100° (*d*) 20° 18.....

19 If each edge of a rectangular solid is doubled, the total area is multiplied by (*a*) 8 (*b*) 2 (*c*) 6 (*d*) 4 19.....

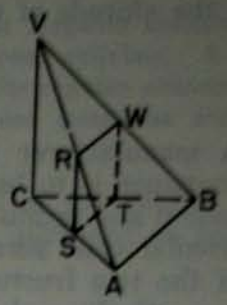
20 The locus of the centers of equal small circles of a sphere is (*a*) a point (*b*) a plane (*c*) two planes (*d*) a spherical surface 20.....

## Part II

**Answer five questions from this part. Show all work unless otherwise directed.**

21 Prove: If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 In the figure, a triangular pyramid $V$-$ABC$ has $VC$ perpendicular to base $ABC$. The midpoints of $VA$, $AC$ and $CB$ are $R$, $S$ and $T$, respectively. The plane through $R$, $S$ and $T$ intersects $VB$ in $W$. Prove: $RSTW$ is a rectangle. [10]




23 Prove: In two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other. [10]

24 Planes $M$ and $N$ are parallel and $k$ distance apart. Point $A$ lies in $M$ and point $B$ lies in $N$.

*a* Describe fully the locus of points at a distance $s$ from $A$. [3]

*b* Describe fully the locus of points equidistant from $A$ and $B$. [3]

*c* *Name* the locus of points common to parts *a* and *b* when $s > \frac{k}{2}$ and $AB$ is perpendicular to $M$. [2]

*d* *Name* the locus of points common to parts *a* and *b* when $s = \frac{k}{2}$ and $AB$ is perpendicular to $M$. [2]

25 An isosceles triangle, each of whose base angles is $\theta$ and whose leg is $a$, is rotated through 180°, using as an axis its altitude to the base.

*a* Show that the volume $V$ of the resulting solid is given by the formula $V = \frac{\pi a^3 \sin \theta \cos^2 \theta}{3}$. [5]

*b* Using the formula given in part *a*, find $V$ to the *nearest integer* if $a = 5.2$ and $\theta = 27°$. [Use the approximation $\pi = 3.14$.] [5]

26 Given a regular tetrahedron whose edge is represented by $a$. Derive a formula for the volume $V$ in terms of $a$. [10]

27 On a sphere of radius 9 inches the perimeter of a spherical triangle is $12\pi$ inches. The sides of the triangle are in the ratio 3 : 4 : 5.

*a* Find the sides of the triangle in degrees. [3]

*b* Find the angles of its polar triangle. [2]

*c* Find the area of the polar triangle in square inches. [Answer may be left in terms of $\pi$.] [3]

*d* A zone on this sphere is equal in area to the polar triangle. Find the number of inches in the altitude of the zone. [2]

28 A manufacturer wishes to change the container in which his product is retailed. The new container is to have the same volume as the old, which was a right circular cylinder with altitude 6 and radius of its base 3. The new container is to be composed of a frustum of a right circular cone surmounted by a frustum of another right circular cone so that the smaller bases of the two frustums coincide. The radii of the bases of the lower frustum are $3\frac{1}{2}$ and 2, respectively, and its altitude is 4. The radius of the upper base of the upper frustum is 3. Find, to the *nearest tenth*, the altitude of the upper frustum. $[V = \frac{\pi h}{3}(r^2 + r_1^2 + rr_1).]$ [10]

# FOR TEACHERS ONLY

# SG

## INSTRUCTIONS FOR RATING
## SOLID GEOMETRY

**Thursday,** January 22, 1959 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2½ credits for each correct answer; allow no partial credit. For questions 15–20, allow credit if the pupil has written the correct answer instead of the letter *a*, *b*, *c* or *d*.

(1) 80

(2) 240

(3) 8

(4) 18

(5) 41

(6) 132

(7) 14

(8) $\frac{500}{3}\pi$

(9) 70

(10) $5\sqrt{3}$ *or* 8.7

(11) never

(12) always

(13) sometimes

(14) always

(15) *b*

(16) *c*

(17) *d*

(18) *c*

(19) *d*

(20) *d*