SOLID GEOMETRY

Wednesday, January 27, 1954—9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Express the area of a sphere in terms of its diameter *d*.   1...............

2. The altitude of a square pyramid is 3 and each lateral face makes an angle of 45° with the base. Find the volume of the pyramid.   2...............

3. The area of a zone on a sphere of radius 2 is 6π. Find the altitude of the zone.   3...............

4. A regular triangular prism has an altitude of 5 inches and a base with each side equal to 2 inches. Find the volume of the prism in cubic inches. [Answer may be left in radical form.]   4...............

5. A cylinder of revolution and a cone of revolution have equal lateral areas and equal bases. If the altitude of the cylinder is 5, find the slant height of the cone.   5...............

6. If the volume of a cube is 3⅜, find its total area.   6...............

7. How many metal spheres, each having a radius of 2 inches, can be made by melting and using all the material from a metal sphere having a radius of 10 inches?   7...............

8. A cone has a base of 25 square feet. A section of the cone formed by a plane parallel to the base and 3 feet from the vertex of the cone is 4 square feet. Find the altitude of the cone in feet.   8...............

9. The angles of a spherical triangle are 70°, 85° and 95°. Find its area in spherical degrees.   9...............

10. The perimeters of the bases of a frustum of a regular pyramid are 15 and 23, and the slant height is 7. Find the lateral area of the frustum.   10...............

11. A lateral edge of a prism is 5 inches and is inclined 62° to the base. Find the altitude of the prism to the *nearest inch*.   11...............

12. The angles of a spherical triangle are 50°, 70° and 80°. Find the number of degrees in the perimeter of the polar triangle.   12............ .

Directions (13-15): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c*.

13. The sum of the sides of a convex spherical polygon may be   (*a*) 300°   (*b*) 360°   (*c*) 420°   13........

14. Two face angles of a trihedral angle are 100° and 150°. The third face angle may be   (*a*) 50°   (*b*) 80°   (*c*) 110°   14........

15. The number of points equidistant from all points on a circle and also at a distance *d* from the plane of the circle is   (*a*) 1   (*b*) 2   (*c*) unlimited   15........

Directions (16-20): For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

16. If two planes are perpendicular to each other, a line drawn from a point in one of them to a point in the other is perpendicular to the other plane.   16................

17. If a line is parallel to a plane, it is parallel to all lines in the plane.   17................

18. If the faces of a polyhedron are regular polygons, the polyhedron is regular.   18................

19. If a line is not in either face of a dihedral angle but is parallel to its edge, then the line is parallel to the faces of the dihedral angle.   19................

20. The area of a lune is to the area of the sphere on which it is drawn as the angle of the lune is to 720°.   20................

Part II

Answer two questions from part II.

21. Prove: If two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other.   [10]

22. Prove: If the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first.   [10]

23. The points *A*, *B* and *C* are on a circle. A plane is passed perpendicular to chord *AB* at its mid-point, and a plane is passed perpendicular to chord *BC* at its mid-point. Prove that the intersection of these planes is a line perpendicular to the plane of the circle at its center.   [10]

24. Given the three points *A*, *B* and *C* on line *l*, point *B* being the mid-point of segment *AC*.
   a. Describe fully the locus of points
      (1) at distance *d* from *B*   [2]
      (2) at distance *r* from *l*   [2]
      (3) equidistant from *A* and *C*   [2]
   b. What is the locus of points satisfying the conditions of both (1) and (3) above?   [2]
   c. What is the locus of points satisfying the conditions of both (2) and (3) above?   [2]

Part III

Answer three questions from part III.

25. The volume of a sphere is 5720 cubic inches.
   a. Find the radius of the sphere to the *nearest tenth of an inch.* [Use $\pi = 3.14$]   [5]
   b. Using the result found in answer to *a*, find to the *nearest square inch* the area of a spherical triangle, on the sphere, whose angles are 68°, 72° and 100°.   [5]