The University of the State of New York

314TH HIGH SCHOOL EXAMINATION

# SOLID GEOMETRY

**Thursday,** January 24, 1952 — 9.15 a. m. to 12.15 p. m., only

## Instructions

**Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.**

**Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.**

**The minimum time requirement is four or five recitations a week for half a school year.**

## Part II

**Answer two questions from part II.**

21 Prove that if a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines. [10]

22 Prove that the sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

23 Prove that the section of a tetrahedron made by a plane parallel to two opposite edges is a parallelogram. [10]

*24 Given plane $P$ and point $M$ not in $P$.

*a* By means of a drawing, represent the locus of points equidistant from $P$ and $M$. [6]

*b* What is the name of the curve formed by plane $q$ intersecting the locus found in answer to *a* if $q$ is (1) parallel to P, (2) perpendicular to $P$? [2, 2]

* This question is based upon one of the optional topics in the syllabus.

## Part III

**Answer three questions from part III.**

25 A laboratory beaker in the form of a right circular cylinder is partially filled with water. The diameter of its base is 5 inches. When a piece of lead is completely immersed in the water, the level of the water rises 2 inches. If lead weighs 6.5 oz. per cu. in., find the weight of the lead to the *nearest pound.* [Use $\pi = \frac{22}{7}$] [10]

26 The lower base edge of a frustum of a regular square pyramid is 15 in. and its slant height is 10 in. A dihedral angle formed by a lateral face and the lower base is 70°.

*a* Find the altitude of the frustum to the *nearest inch.* [2]

*b* Find the upper base edge to the *nearest inch.* [5]

*c* Find the volume of the frustum to the *nearest 10 cubic inches.*
[$V = \frac{1}{3}h\,(B_1 + B_2 + \sqrt{B_1B_2})$] [3]

27 A zone and a spherical quadrilateral drawn on the same sphere are equal in area. The radius of the sphere is 12 in. and the angles of the quadrilateral are 150°, 65°, 100° and 95°. Find the altitude of the zone. [10]

28 A solid consists of a right circular cylinder and hemispherical caps that fit exactly on the bases of the cylinder. The length of the cylinder is one half the total length $l$ of the solid. Express the volume of the solid as a function of $l$. [Answer may be left in terms of $\pi$.] [10]

**Fill in the following lines:**

Name of pupil.....Name of school.....

## Part I

**Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.**

1 A lateral edge of an oblique prism is 15 in. and the perimeter of a right section is 24 in. Find in square inches the lateral area of the prism. 1.....

2 The altitude of a pyramid is 16 in. A plane parallel to the base forms a section whose area is one fourth the area of the base. Find the distance of this plane from the vertex. 2.....

3 Find the lateral area of a right circular cone whose altitude is 12 and whose radius is 5. [Answer may be left in terms of $\pi$.] 3.....

4 The locus of points at a given distance $d$ from a given sphere whose radius is $r$ $(d < r)$ is $(a)$ two circles $(b)$ one sphere $(c)$ two spheres. Which is correct $(a)$, $(b)$ or $(c)$? 4.....

5 How many spherical degrees are there in a lune whose angle is 1°? 5.....

6 The altitudes of two zones on the same sphere are in the ratio 2:3. Find the ratio of their areas. 6.....

7 The volume of a circular cone is $V$. Express in terms of $V$ the volume of a cone whose radius is twice the radius of the given cone and whose altitude is one half the altitude of the given cone. 7.....

Directions (questions 8–10) — If the blank in each of the following statements is filled by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that correctly completes *each* statement and write this word on the line at the right.

8 A section of an oblique prism made by a plane parallel to a lateral edge is ... a rectangle. 8.....

9 The locus of points at a given distance $d$ from a given line and also equally distant from two given points on this line is ... a circle. 9.....

10 Lines $l$ and $l'$ are perpendicular. A dihedral angle formed by a plane containing $l$ and a plane containing $l'$ is ... a right dihedral angle. 10.....

11 A regular triangular prism has a height of 12 in. and a base with one side equal to 5 in. Find its volume in cubic inches. [Answer may be left in radical form.] 11.....

12 A line segment 16 in. long is inclined 60° to a plane. Find the length of its projection on the plane. 12.....



[OVER]

13 A spherical triangle has angles of 90°, 60° and 90°. What fractional part of the area of the sphere is the area of the triangle? 13.....

14 The total area of a cube is 18 sq. in. Find the length of one of the diagonals of the cube. 14.....

15 The lateral area of the frustum of a regular hexagonal pyramid is 390. The base edges are 8 and 5. Find the slant height. 15.....

16 Find the number of faces of a regular polyhedron that has 4 vertices and 6 edges. 16.....

17 The area of a great circle of a sphere is $A$. Express the area of the sphere in terms of $A$. 17.....

18 Spherical triangles $ABC$ and $A'B'C'$ are polar triangles. The pole of $A'C'$ is $B$. Find the number of degrees in spherical angle $B$ if arc $A'C' = 70°$. 18.....

19 The volumes of two similar pyramids are in the ratio 8 to 125. Find the ratio of their total areas. 19.....

20 Two face angles of a trihedral angle are 70° and 150°. The third face angle may be (*a*) 60° (*b*) 100° (*c*) 140°. Which is correct, (*a*), (*b*) or (*c*)? 20.....