The University of the State of New York

308TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 26, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

22 If two planes are perpendicular to each other, a line perpendicular to one of them is parallel to the other. [10]

23 Prove that if a plane divides the lateral edges of a pyramid proportionally, the plane is parallel to the base of the pyramid. [10]

24 Prove that if two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other. [10]

Part III

Answer three questions from part III.

25 Given trapezoid $ABCD$ with angles A and B right angles. DA is 6 inches, AB is 4 inches and BC is 9 inches. The trapezoid is revolved through 360° about BC as an axis. Express, in terms of π, (*a*) the total area of the resulting solid, (*b*) the volume of the resulting solid. [5, 5]

26 Find the area of a spherical triangle on a sphere whose radius is 7 inches, if the perimeter of its polar triangle is 180 degrees. [Use $\pi = \frac{22}{7}$] [10]

27 The slant height of a frustum of a regular square pyramid makes with the lower base an angle A. The lower base edge is a and the upper base edge is b. Show that the lateral area S of the frustum is given by the formula: $S = \frac{a^2 - b^2}{\cos A}$ [10]

28 The volume of a sphere is 122 cu. in.

a Using logarithms, find, to the *nearest tenth,* the radius of the sphere. [Use $\pi = 3.14$] [7]

b Using the result obtained in answer to *a,* find, to the *nearest integer,* the area of the sphere. [3]

Fill in the following lines:

Name of pupil.....Name of school .....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

Directions (questions 1–5) — Write on the line at the right of *each* statement the word or number which, when inserted in the blank, will make the statement true.

1 Two planes perpendicular to the same ... are parallel to each other. 1.....

2 If a point is 6 inches from each face of a dihedral angle and 12 inches from the edge of the angle, the dihedral angle contains ... degrees. 2.....

3 Two face angles of a trihedral angle are 80 degrees and 110 degrees. The third face angle must be greater than 30 degrees and less than ... degrees and may have any value between these two limits. 3.....

4 A plane bisects a sphere whose radius is 5 inches. The locus of points 2 inches from the plane and 1 inch from the sphere consists of ... circles. 4.....

5 If one side of a spherical triangle contains 80 degrees, the angle opposite this side in the polar triangle contains ... degrees. 5.....

6 Find the volume of a regular hexagonal prism whose base edge is 6 and whose altitude is 10. [Answer may be left in radical form.] 6.....

7 Find the length of a diagonal of a cube whose total surface area is 24 square inches. [Answer may be left in radical form.] 7.....

8 Express the volume of a regular square pyramid in terms of its altitude h and its base edge e. 8.....

9 The total area of a regular tetrahedron is $4\sqrt{3}$. Find an edge. 9.....

10 The volumes of two similar cylinders of revolution are in the ratio 1 : 8. Find the ratio of their total areas. 10.....

11 Find the lateral area of a frustum of a right circular cone the radii of whose bases are 6 and 8 and whose slant height is 10. [Answer may be left in terms of π.] 11.....

12 The radius of a sphere is 13 inches. Find the area of a small circle whose plane is 5 inches from the center of the sphere. [Answer may be left in terms of π.] 12.....

13 The area of a zone of a sphere is 120 π and its altitude is 5. Find the radius of the sphere. 13.....

14 If the sum of three angles of a spherical quadrilateral is 270 degrees, the fourth angle must be (*a*) less than 90 degrees, (*b*) equal to 90 degrees, (*c*) greater than 90 degrees. [Answer *a*, *b* or *c*.] 14.....

15 Find the number of degrees in the angle of a lune whose area is 100 spherical degrees. 15.....

16 Two angles of a spherical triangle are 100 degrees and 90 degrees and the area of the triangle is 60 spherical degrees. Find the number of degrees in the third angle of the triangle. 16.....

[OVER]

Directions (questions 17–20) — If the blank in each statement is replaced by one of the words *always, sometimes,* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

17 A line perpendicular to a line in a plane is ... perpendicular to the plane. 17.....

18 A line segment oblique to a plane is ... greater than its projection on the plane. 18.....

19 Two lines parallel to the same plane are ... parallel to each other. 19.....

20 Two diagonals of a rectangular parallelepiped are ... perpendicular to each other. 20.....