The University of the State of New York

299TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 30, 1947 — 9.15 a. m., to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that the sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

22 Two parallel line segments AB and CD are oblique to plane P. The plane of AB and CD is not perpendicular to P. Prove that the projections of AB and CD on P are parallel. [10]

23 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

24 Given plane P and line l of unlimited length.

a Describe fully the locus of points 5 inches from plane P. [4]

b Describe fully the locus of points 3 inches from line l. [3]

c Tell whether *each* of the following statements is *true* or *false*:

(1) If l is parallel to P and 6 inches from P, the locus of points satisfying both conditions given in *a* and *b* consists of two parallel lines. [1]

(2) If l is parallel to P and 8 inches from P, there are no points satisfying both conditions given in *a* and *b*. [1]

(3) If l is perpendicular to P, the locus of points satisfying both conditions given in *a* and *b* consists of two circles whose planes are parallel. [1]

Part III

Answer three questions from part III.

25 A solid has the form of a right circular cone whose radius is 3 feet and whose altitude is 4 feet. When the solid, with vertex down, is placed in water, it sinks to a depth of $2\frac{1}{2}$ feet.

a What is the volume of that part of the solid which is under water? [Answer may be left in terms of π.] [7]

b Find the weight of the solid. [The weight of the solid is equal to the weight of the water displaced. Consider the water to weigh 62.5 lb. per cu. ft.] [Answer may be left in terms of π.] [3]

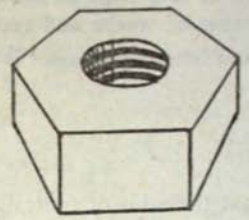
26 A spherical triangle whose angles are 100°, 70° and 50° is drawn on a sphere whose radius is 6 inches.

a Find in square inches the area of the spherical triangle. [Answer may be left in terms of π.] [5]

b Find the number of degrees in the angle of a lune on the same sphere, equal in area to the given triangle. [3]

c Find the altitude of a zone on the same sphere, equal in area to the given triangle. [2]

27 The figure at the right represents a regular hexagonal nut. The radius of the hole is $\frac{1}{2}$ inch. The edge of the nut is 1 inch and its thickness is $\frac{1}{2}$ inch. Find, correct to the *nearest tenth of a cubic inch*, the volume of the material in the nut. [Use $\pi = 3.14$] [10]



28 The base of a regular pyramid is a square. The lateral edge a makes with the base edge an angle x.

a Show that the formula for the lateral area S of the pyramid is $S = 4a^2 \cos x \sin x$. [6]

b Find the value of S, correct to the *nearest integer*, when $a = 3.5$ and $x = 43°$. [4]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–4) — Write on the line at the right of *each* question the expression which, when inserted in the blank, will make the statement true.

1 A line segment has its longest projection on a plane when it is ... to the plane. 1.....

2 Any section of a cone made by a plane containing an element is .... 2.....

3 Two face angles of a trihedral angle are 100° and 70°. The third face angle must be greater than ... degrees and less than ... degrees and may have any value between these limits. 3.....

4 The area of a circle formed by passing a plane 5 inches from the center of a sphere whose radius is 13 inches is .... [Answer may be left in terms of π.] 4.....

Directions (questions 5–9) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

5 The diagonals of a parallelepiped are ... equal. 5.....

6 If a pyramid is cut by a plane parallel to its base, the section formed is ... similar to the base. 6.....

7 If a line is perpendicular to a line in a plane, it is ... perpendicular to the plane. 7.....

8 Two lines perpendicular to the same line are ... parallel. 8.....

9 If lines l and l' lie in two parallel planes, then l and l' are ... parallel to each other. 9.....

Directions (questions 10–11) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

10 A given point on a sphere is the pole of only one great circle of the sphere. 10.....

11 A plane perpendicular to the faces of a dihedral angle is perpendicular to the edge of the angle. 11.....

12 The areas of two similar solids are in the ratio 1:4. Find the ratio of the volume of the smaller solid to the volume of the larger. 12.....

13 Find an edge of a regular tetrahedron whose total area is $36\sqrt{3}$ 13.....

14 A cone and a cylinder have equal radii and equal altitudes. Find the ratio of the volume of the cone to the volume of the cylinder. 14.....

15 Express in terms of π the volume of a sphere whose radius is 6. 15.....

16 Express the total area of a right circular cylinder of radius r and altitude $2r$. 16.....

17 The base of a right prism is a rhombus whose diagonals are 12 and 16. The altitude of the prism is 20. Find its lateral area. 17.....

18 Find the edge of a cube if its diagonal is $4\sqrt{3}$ 18.....

19 Find the lateral area of a frustum of a right circular cone whose slant height is 8 and whose upper and lower bases have radii of 6 and 8 respectively. [Answer may be left in terms of π.] 19.....

20 The angle of a lune is 40°. Find its area in spherical degrees. 20.....