The University of the State of New York

296TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 31, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you' may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other plane. [10]

22 Prove that in two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other. [10]

23 *a* Line *a* is perpendicular to the plane of a circle at its center O. Prove that any point on *a* is equidistant from all points on the circle. [4]

  b Point P is equidistant from all points on a circle whose center is O. Prove that line PO is perpendicular to the plane of the circle. [4]

  c Complete the following statement: The locus of points equidistant from all points on a given circle is .... [2]

Part III

Answer three questions from part III.

24 A bar of metal has the form of a regular square prism whose dimensions are 5 in. by 5 in. by 7 ft. The bar is to be melted and cast into solids each having the form of a hollow cylinder whose length is 2 in., whose outer radius is 1 in., and whose inner radius is $\frac{3}{4}$ in. Allowing 10 cu. in. for waste, how many solids can be made from the bar? [Use $\pi = \frac{22}{7}$] [10]

25 The base of an isosceles triangle is b and one of the equal base angles is θ. Express in terms of b and θ:

  a The altitude on the base of the triangle [2]

  b One of the equal sides of the triangle [2]

  c The surface and the volume of the solid formed by revolving the triangle through 360° about b as an axis [3, 3]

[1]

[OVER]

26 A metal casting has the form of a frustum of a right circular cone capped by a hemisphere. The radius of the hemisphere and the radius of the upper base of the frustum are each 2 in.; the radius of the lower base and the slant height of the frustum are each 5 in. Allowing .21 lb. per cu. in., find, correct to the *nearest pound*, the weight of the casting. [The volume of a frustum of a right circular cone is given by the formula $V = \dfrac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$]

[Use $\pi = {}^{2}\!/_{7}$] [10]

27 The base of a regular pyramid is a pentagon whose side is a; the altitude of the pyramid is h.

    a Express the volume V of the pyramid as a function of a and h. [7]

    b Find V, correct to the *nearest cubic foot*, if $a = 1$ ft and $h = 6$ ft. [3]

*28 The accompanying figure represents a special type of prismatoid. Base $ABCD$ is a rectangle, faces BCF and ADE are isosceles triangles, and faces $ABFE$ and $DCFE$ are isosceles trapezoids. AB is 16, BC is 10, EF is 6 and FH, the altitude of triangle BCF, is 13.

Using the formula $V = \dfrac{h}{6} (B + B' + 4m)$,

find the volume of the prismatoid. [10]
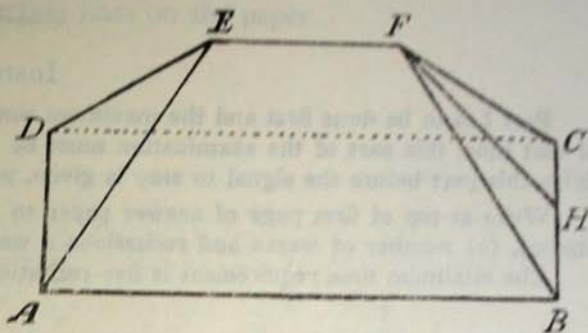
* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school...Name of pupil..

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Corresponding altitudes of two similar parallelepipeds are in the ratio 2:3. Find the ratio of their volumes.

1................

2 A rectangle whose sides are a and b is revolved through 360° about b as an axis. Find the lateral area of the solid thus formed. [Answer may be left in terms of π.]

2................

3 The base edge of a regular triangular prism is 2 and its lateral edge is 8. Find the total area of the prism. [Answer may be left in radical form.]

3................

4 The perimeter of the midsection of a frustum of a regular pyramid is 12 and the slant height is 3. Find the lateral area of the frustum.

4................

5 A plane is passed parallel to the base of a pyramid so that the area of the section thus formed is $\frac{4}{9}$ the area of the base. If the altitude of the pyramid is 12 in., how far from the vertex of the pyramid is this plane?

5................

6 Find the area of a sphere whose radius is 5. [Answer may be left in terms of π.]

6................

7 A zone is drawn on a sphere whose radius is 7. The altitude of the zone is 2. Find the area of the zone. [Answer may be left in terms of π.]

7................

8 The area of a lune is 60 spherical degrees. How many degrees are there in the angle of the lune?

8................

9 The angles of a spherical triangle are 80°, 70°, and 65°. How many spherical degrees are there in the area of the triangle?

9................

10 The edge of a cube is 10 inches. If the edge of the cube is increased by 1 in., the volume of the cube is increased by approximately (a) 3 cu. in., (b) 30 cu. in. or (c) 300 cu. in. Which is correct (a), (b) or (c)?

10................

11 The circumference of a great circle of a sphere is twice that of a small circle of the sphere. If the radius of the sphere is 2 in., the distance from the center of the sphere to the plane of the small circle is (a) $\sqrt{2}$ in., (b) $\sqrt{3}$ in. or (c) 1 in. Which is correct (a), (b) or (c)?

11................

12 The altitude of a cone of revolution is 8 and the radius of its base is 5. Find, correct to the *nearest degree*, the angle at which an element of the cone is inclined to the base.

12................

13 Does the following statement *define* a regular polyhedron? A regular polyhedron is a polyhedron whose faces are congruent regular polygons. [Answer *yes* or *no*.]

13................

14 Is a plane determined if it passes through a given point and is perpendicular to a given plane? [Answer *yes* or *no*.]

14................

[3]

[OVER]

Directions (questions 15–20) — If in each of the following statements the blank space is replaced by one of the words *always, sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

15 If two planes are parallel to the same line, they are ... parallel to each other.

16 The projection of an isosceles triangle on a plane is ... an isosceles triangle.

17 The sum of the sides of any convex spherical polygon is ... greater than 360°.

18 The sum of the angles of any convex spherical polygon of n sides is ... greater than $(n - 2)$ 180°.

19 If three straight lines pass through the same point, each of these lines is ... perpendicular to the other two.

20 The locus of points equally distant from two parallel planes and at a given distance from a given line ... consists of two parallel lines.

15..................

16..................

17..................

18..................

19..................

20..................