The University of the State of New York

293D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 25, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that the sum of any two face angles of a trihedral angle is greater than the third face angle. [10]

22 *a* Prove that any point in the line which is perpendicular to the plane of a circle at its center is equidistant from all points on the circle. [6]

b State the converse of the theorem given in *a*. [3]

c Is the theorem given in *a* or the converse as you have given it in answer to *b*, when taken alone, sufficient to prove that the locus of points equidistant from a given circle is the line perpendicular to the plane of the circle at its center? [Answer *yes* or *no*.] [1]

23 Prove that a spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

*24 *a* Write a brief statement explaining Cavalieri's theorem. [5]

b Explain briefly how the theorem is used to prove that two pyramids having equal bases and equal altitudes are equal. [5]

* This question is based on one of the optional topics in the syllabus.

Part III

Answer three questions from part III.

25 A rectangular sheet of metal is $5\frac{1}{2}$ ft long, 2 ft wide and $\frac{1}{8}$ in. thick. How many spherical shot $\frac{1}{2}$ in. in diameter can be cast from this sheet of metal? [Use $\pi = \frac{22}{7}$] [10]

26 Rectangle $ABCD$ is revolved through 360° first about AB as an axis and then about BC as an axis. Show that the ratio of the volumes of the solids thus formed is equal to the ratio of their total areas. [10]

27 The area of an equilateral spherical triangle drawn on a sphere whose radius is 7 in. is $25\frac{2}{3}$ sq. in. Find the angles of the triangle. [Use $\pi = \frac{22}{7}$] [10]

28 The slant height s of a right circular cone makes with the base an angle θ.

a Express the volume V of the cone as a function of s and θ. [5]

b Find, correct to the *nearest tenth,* the value of V if $s = 2.80$ and $\theta = 67°$. [Use $\pi = 3.14$]
[*Suggestion:* Compute by means of logarithms.] [5]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The altitude of a cone of revolution is 12 and the radius of its base is 5. Find the lateral area of the cone. [Answer may be left in terms of π.] 1.....

2 Express the volume of a regular triangular pyramid in terms of its base edge e and its altitude h. [Answer may be left in radical form.] 2.....

3 Express the lateral area of a frustum of a regular pyramid in terms of its slant height l and the perimeter p of its midsection. 3.....

4 The area of a sphere is 616 square inches. Find its diameter. [Use $\pi = \frac{22}{7}$]. 4.....

5 A zone whose altitude is 2 is drawn on a sphere whose radius is 8. Find the area of the zone. [Answer may be left in terms of π.] 5.....

6 How many spherical degrees are there in a lune whose angle is 10°? 6.....

7 The sides of a spherical triangle are 78°, 85° and 97°. Find the spherical excess of its polar triangle. 7.....

Directions (questions 8–13) — If the blank in *each* statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.

8 If a line is parallel to one of two intersecting planes, it is ... parallel to the other. 8.....

9 If two planes are perpendicular to each other, a line perpendicular to one of them at any point in their intersection will ... lie in the other. 9.....

10 The projection of a square on a plane is ... a rectangle. 10.....

11 If the sides of one acute angle are respectively parallel to the sides of another acute angle, the angles are ... equal. 11.....

12 The locus of points on a given sphere which are a given distance from a given plane ... consists of two circles. 12.....

13 If two angles of a spherical triangle are complementary, the triangle is ....a right spherical triangle. 13.....

Directions (questions 14–17) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

14 More than one common perpendicular can be drawn to two lines if the lines (*a*)intersect (*b*)are parallel (*c*)are skew 14.....

15 If two cones of revolution are similar and if the total area of the first is twice that of the second, the ratio of the volume of the first to the volume of the second is (*a*) $\sqrt{2}:1$ (*b*)$2\sqrt{2}:1$ (*c*)8:1 15.....

16 If the faces of a regular polyhedron are equilateral triangles, the polyhedron may have (*a*)six faces (*b*)eight faces (*c*)twelve faces 16.....

17 A plane is passed parallel to and 3 inches from the base of a pyramid whose altitude is 8 inches. The ratio of the area of the section thus formed to the area of the base is (*a*)9:64 (*b*)5:8 (*c*)25:64 17.....

[OVER]

Directions (questions 18–20) — Indicate whether *each* statement is true or false by writing the word *true* or the word *false* on the line at the right.

18 If the section of a prism made by a plane parallel to one of its lateral edges is a rectangle, the prism must be a right prism. 18.....

19 Diagonals of a cube are perpendicular to each other. 19.....

20 The dihedral angle formed by any two faces of a regular tetrahedron is equal to 60°. 20.....