The University of the State of New York

290TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 20, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other. [10]

22 One side of a right angle is parallel to a plane. If the plane of the angle is not perpendicular to the given plane, prove that the projection of the angle on the given plane is a right angle. [10]

23 Prove that every section of a sphere made by a plane is a circle. [10]

*24 Given plane M and point P which is 8 inches from M

    a By means of a drawing, represent the locus of points equidistant from plane M and point P. [4]

    b On the drawing made in answer to *a*, represent also the locus of points d inches from M. [3]

    c Describe fully the locus of points which satisfy the conditions in both *a* and *b*, if $d = 5$ inches. [3]

Part III

Answer three questions from part III.

Use $\pi = \frac{22}{7}$ unless otherwise stated.

25 ABC is a spherical triangle drawn on a sphere whose center is O. The face angles of trihedral angle O-ABC are 75°, 60° and 120°.

    a Find the number of degrees in *each* side of triangle ABC. [3]

    b If the radius of the sphere is 10, find, correct to the *nearest integer,* the area of the polar triangle of ABC. [7]

* This question is based on one of the optional topics in the syllabus.

[1] [OVER]

26 A casting is in the form of a frustum of a right circular cone containing a hollow cylindrical core whose axis coincides with the axis of the frustum. The radii of the upper and lower bases of the frustum are 3 inches and 5 inches respectively, its height is 6 inches and the diameter of the hollow core is 2 inches. Find, correct to the *nearest 10 pounds,* the weight of the casting if the metal used weighs .28 pounds per cubic inch. The volume of a frustum of a circular cone is given by the formula $V = \frac{1}{3} \pi h(r_1{}^2 + r_2{}^2 + r_1 r_2)$ [10]

27 A sector of a circle of radius 12 is rolled up to form a cone, the arc of the sector forming the circle of the base of the cone. If the angle of the sector contains 150°, find the volume of the cone. [Answer may be left in radical form.]   [10]

28 A circle inscribed in square $ABCD$ is tangent to sides AB, BC, CD, DA at points X, Y, Z, W respectively. Diameter XZ and diagonal AC are drawn.

    a If the whole figure is revolved through 360° around XZ as an axis, what solid is generated by (1) circle $XYZW$, (2) square $ABCD$, (3) diagonal AC?   [1, 1, 2]

    b Show that the volume generated by circle $XYZW$ equals the difference between the volumes of the solids generated by square $ABCD$ and diagonal AC.   [6]

Fill in the following lines:

Name of school..Name of pupil...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If two straight lines are tangent to a sphere at the same point, is their plane tangent to the sphere? [Answer *yes* or *no.*] 1................

2 Find the lateral area of a right circular cylinder circumscribed about a sphere whose radius is 3. [Answer may be left in terms of π.] 2................

3 Find the lateral area of a right circular cone whose altitude is 4 and whose slant height is 5. [Answer may be left in terms of π.] 3................

4 Express the area K of a lune in terms of its angle A and the radius r of the sphere on which it is drawn. 4................

5 A line segment 20 inches long is inclined to a plane at an angle of 60°. Find the length of the projection of the line segment on the plane. 5................

6 The area of the base of a pyramid 10 inches high is 200 square inches. Find the area of the section of the pyramid cut off by a plane parallel to the base and 4 inches from it. 6................

7 Find the lateral area of the frustum of a right circular cone the radii of whose bases are 8 and 5 and whose slant height is 6. [Answer may be left in terms of π.] 7................

8 How many pyramids are formed by drawing lines from the center of one face of a cube to all of its vertices? 8................

9 The radius of a sphere is 13 inches. Find the radius of a circle formed by a plane passing through the sphere 5 inches from its center. 9................

10 A wooden model of a steel casting weighs 24 pounds. If the weights of the wood and the steel are 28 and 490 pounds per cubic foot respectively, find the weight of the casting. 10................

11 Which, if any, of the following statements is false?

  a A plane section of a right circular cylinder may be a parallelogram.

  b A plane section of a right circular cone may be an ellipse.

  c A plane section of a triangular pyramid may be a hexagon. 11................

Directions (questions 12–15) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c*.

12 The number of straight lines equidistant from two intersecting planes and at a given distance from their intersection is (*a*) 1, (*b*) 2, (*c*) 4. 12................

13 VO is the altitude of a regular pyramid V-$ABCD$. P is any point in side BC of its base. As point P moves from point B to point C, angle OPV (*a*) first increases and then decreases, (*b*) first decreases and then increases, (*c*) remains constant 13................

14 The angles of a spherical triangle may be (*a*) 70°, 60°, 40°; (*b*) 200°, 300°, 50°; (*c*) 80°, 50°, 100° 14................

15 If the areas of two zones drawn on the same sphere are in the ratio 1:2, the ratio of their altitudes is (*a*) 1:2, (*b*) 1:4, (*c*) 1:8 15................

Directions (questions 16–20) — Indicate whether *each* statement is true or false by writing the word *true* or the word *false* on the line at the right.

16 A plane may be parallel to each of two skew lines.

16................

17 Lines l and r are perpendicular to each other; M is any plane through l; then r is always perpendicular to M.

17................

18 If two planes intersect, a line in one of the planes perpendicular to the intersection is always perpendicular to the other.

18................

19 The perimeter of a convex spherical polygon drawn on a sphere whose radius is 5 must be less than 10π.

19................

20 If the base of a pyramid is a parallelogram, the plane determined by the vertex of the pyramid and a diagonal of the base will divide the original pyramid into two equal pyramids.

20................