The University of the State of New York

286TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 21, 1943 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines. [10]

22 Prove that if the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

23 Planes P and Q are perpendicular to each other and intersect in line RS. Line segment AB is parallel to P and its projection on Q is line segment MN.

a Prove that plane AB-MN is parallel to plane P. [8]

b Prove that MN is parallel to RS. [2]

*24 *a* State Cavalieri's theorem. [3]

b Make a drawing to illustrate the theorem. [5]

c Write a description of the drawing and explain how it illustrates the theorem. [2]

Part III

Answer three questions from part III.

25 The sides of a spherical triangle drawn on a sphere whose radius is 3.5″ are 55°, 60° and 75°.

a Find the number of degrees in the angles of its polar triangle. [3]

b Find, correct to the *nearest square inch*, the area of the polar triangle. [Use $\pi = \frac{22}{7}$] [7]

26 A solid column of wood has the form of a regular hexagonal prism whose base edge is e and whose altitude is h.

a Express the volume V of the column in terms of e and h. [5]

b If $e = 1.0'$ and $h = 25.0'$, find, correct to the *nearest pound*, the weight of the column, allowing 16 pounds per cubic foot. [5]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express the diagonal d of a cube in terms of its edge e. 1.....

2 Two parallelepipeds have equal bases. The altitude of the smaller is 2 feet and that of the larger is 3 feet. What is the ratio of the volume of the smaller parallelepiped to the volume of the larger? 2.....

3 Find the lateral area of a frustum of a regular triangular pyramid whose base edges are 12 and 4 and whose slant height is 5. 3.....

4 The slant height of a regular square pyramid is 6 and it is inclined to the base at an angle of 60°. Find the lateral area of the pyramid. 4.....

5 If the faces of a polyhedron are regular polygons, is the polyhedron always a regular polyhedron? [Answer *yes* or *no*.] 5.....

6 If the radius of the base of a circular cone is doubled and the altitude is divided by 2, the volume of the cone (*a*) is increased, (*b*) is decreased or (*c*) remains the same. Which is true, (*a*), (*b*) or (*c*)? 6.....

7 If the altitudes of two similar cylinders of revolution are 3 and 5, what is the ratio of the lateral area of the smaller cylinder to the lateral area of the larger? 7.....

8 If two face angles of a trihedral angle are 50° and 70°, between what two limits must the third face angle lie? 8.....

9 Find the radius of a sphere whose volume is 36π. 9.....

10 A sphere is inscribed in a right circular cylinder the radius of whose base is 3. Find the area of the sphere. [Answer may be left in terms of π.] 10.....

11 The area of a zone drawn on a sphere whose radius is 4 is 16π. Find the altitude of the zone. 11.....

12 Is the perimeter of any convex spherical polygon always less than a great circle of the sphere on which the polygon is drawn? [Answer *yes* or *no*.] 12.....

13 Approximately what portion of the earth's surface is included between the meridians of 12° east longitude and 18° west longitude? 13.....

14 Two cities in the same longitude have latitudes of 20° north and 65° north. Find the distance between the two cities. [Consider the radius of the earth as 3990 miles and use $\pi = \frac{22}{7}$] 14.....

Directions (questions 15–20) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

15 If a plane is perpendicular to the edge of a dihedral angle, it is ... perpendicular to both faces of the angle. 15.....

16 If two line segments included between two planes are equal and parallel, the planes are ... parallel. 16.....

17 If a circular cone is cut by a plane parallel to its base, the area of the section thus formed ... is to the area of the base as the square of its distance from the vertex is to the square of the altitude of the cone. 17.....

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18 The locus of points equidistant from two given intersecting planes and at a given distance from their line of intersection ... consists of four parallel lines. 18.....

19 The locus of points equidistant from two parallel planes and at a given distance from a given point ... consists of two circles. 19.....

20 Two spherical triangles are ... congruent if the three angles of one are equal to the three angles of the other. 20.....

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27 Given trapezoid $ABCD$ with angles A and B right angles. $DA = 6''$, $AB = 4''$ and $BC = 9''$. The trapezoid is revolved through 360° about BC as an axis.
Express in terms of π:

a The total area of the resulting solid [5]

b The volume of the resulting solid [5]

28 A base edge e of a regular square pyramid forms an angle θ with a lateral edge. Show that the volume V of the pyramid is given by the formula $V = \frac{e^3}{6}\sqrt{\tan^2\theta - 1}$ [10]