The University of the State of New York

SOLID GEOMETRY

Thursday, January 22, 1942 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The area of a right section of an oblique prism is 8 square inches and a lateral edge of the prism is 3 inches. Find the number of cubic inches in the volume of the prism.

1.................

2 Two zones are drawn on a given sphere. If the altitude of the first zone is h and of the second is h', what is the ratio of the area of the first zone to the area of the second?

2.................

3 The slant height of a frustum of a cone of revolution is l and the radii of its bases are r and r'; express the lateral area of the frustum in terms of l, r and r'.

3.................

4 A lune is drawn on a sphere whose radius is 6. If the angle of the lune is 15°, find its area. [Answer may be left in terms of π.]

4.................

5 The angles of a spheric triangle are 60°, 70° and 100°. Find the number of spheric degrees in the triangle.

5.................

6 If the ratio of the volumes of two similar cones of revolution is 8 : 1, what is the ratio of their areas?

6.................

7 The radius of the base of a circular cone is three times that of a circular cylinder and their volumes are equal. What is the ratio of the altitude of the cone to the altitude of the cylinder?

7.................

8 If a polyhedron has four vertices, how many edges must it have?

8.................

Directions (questions 9–15) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always*, *sometimes* or *never* on the dotted line at the right.

9 Through a given point in a given line one plane and only one can be drawn perpendicular to the given line.

9.................

10 The locus of points equidistant from two given parallel planes and at a given distance from a given line which is perpendicular to one of the given planes is a pair of parallel lines.

10.................

11 If two face angles of a trihedral angle are 30° and 50°, the third face angle must be less than 80° and greater than 20° and may have any value between these limits.

11.................

12 The bases of a frustum of a pyramid are regular polygons.

12.................

13 Two prisms are congruent if they have congruent bases and equal altitudes.

13.................

14 In a spheric triangle an angle and the side opposite this angle are supplementary.

14.................

15 If two lines are cut by three planes so that the corresponding segments are proportional, the planes are parallel.

15.................

Directions (questions 16–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

16 The lateral area of a cylinder of revolution is equal to the lateral area of a cone of revolution and the radii of their bases are equal. The altitude of the cylinder is equal to (*a*) the slant height of the cone, (*b*) one half the altitude of the cone or (*c*) one half the slant height of the cone.   16.................

17 If two angles not in the same plane have their sides respectively parallel, (*a*) the angles are equal, (*b*) the angles are supplementary or (*c*) the planes of the angles are parallel.   17.................

18 If a pyramid is cut by a plane parallel to its base and one third the distance from the vertex to the base, then the area of the resulting section is to the area of the base as (*a*) 1:3, (*b*) 1:4 or (*c*) 1:9   18.................

19 If line *r* is parallel to line *s* and *s* is perpendicular to line *t*, and *r*, *s* and *t* are not in the same plane, then (*a*) *r* is perpendicular to *t*, (*b*) the plane determined by *s* and *t* is parallel to *r* or (*c*) *t* is perpendicular to the plane determined by *r* and *s*.   19.................

20 If two parallel lines intersect a plane, (*a*) their projections on the plane are always parallel, (*b*) their projections on the plane always coincide or (*c*) the lines are always equally inclined to the plane.   20.................

See instructions for parts II and III on page 1.

Part II

Answer two questions from this part.

21 Prove that through a given point outside a given plane a plane can be passed parallel to the given plane. [10]

22 Prove that the locus of points equidistant from two given points is the plane perpendicular to the line segment joining them at its mid-point. [10]

23 Prove that the sum of the angles of a convex spheric polygon of n sides is greater than $(n-2)180°$. [10]

*24 a State Cavalieri's Principle. [3]

    b Using Cavalieri's Principle, prove that two triangular pyramids having equal bases and equal altitudes are equal. [7]

Part III

Answer three questions from this part.

25 A sphere is circumscribed about a cube whose edge is e.

    a Express in terms of e the area S and the volume V of the sphere. [6]

    b If e is multiplied by 2, by what number is V multiplied? [2]

    c If the surface of the cube is multiplied by 2, by what number is S multiplied? [2]

26 A watering trough has the form of a right prism whose bases are isosceles triangles. The top of the trough is a rectangle 7 feet by 16 inches and is in a horizontal position. If the depth of the trough is 18 inches, find, correct to the *nearest gallon*, the amount of water in the trough when it is filled to a depth of 12 inches. [1 gal. = 231 cu. in.] [10]

27 Each of the following statements is sometimes true and sometimes false. In each case give a condition under which it is true.

    a Through one of two given lines one plane and only one can be passed parallel to the other line. [2]

    b Two trihedral angles are congruent if the three face angles of one are equal respectively to the three face angles of the other. [2]

    c A section of a cylinder is a parallelogram. [2]

    d The volume of a parallelepiped is equal to the product of its three dimensions. [2]

    e The locus of points at a given distance d from a given sphere whose radius is r is a sphere concentric with the given sphere and having a radius $r + d$. [2]

28 $V-ABCD$ is a pyramid whose base is a rhombus. Diagonals AC and BD intersect in O and VO is perpendicular to the base. If $AB = 6.0$ inches, angle $BAD = 30°$ and angle $VBO = 45°$, find, correct to the *nearest cubic inch*, the volume of the pyramid. [10]

* This question is based on one of the optional topics in the syllabus.