The University of the State of New York

280TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 23, 1941 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–11) — Write on the dotted line at the right the expression which, when inserted in the corresponding blank, will make the statement true.

1 Through a given point outside a given plane one and only one line can be drawn ... to the plane. 1.....

2 A line segment makes an angle of 45° with a plane. If its projection on the plane is 8 inches, then the length of the line segment is ... inches. [Answer may be left in radical form.] 2.....

3 If the four diagonals of a right parallelepiped are equal, then the parallelepiped must be a ... parallelepiped. 3.....

4 Two prisms having equal bases are to each other as their .... 4.....

5 If the lateral areas of two similar cones of revolution are in the ratio 4:1, the radius of the larger cone is ... times the radius of the smaller. 5.....

6 The total area T of a cylinder of revolution whose altitude is h and whose radius is r is given by the formula $T =$ .... 6.....

7 The lateral area of a cone of revolution whose slant height is 6 feet and whose radius is 2 feet is ... square feet. [Answer may be left in terms of π.] 7.....

8 If the slant height of a frustum of a regular pyramid is 8 inches, the perimeter of the lower base 10 inches and the perimeter of the upper base 6 inches, the lateral area of the frustum is ... square inches. 8.....

9 If the three angles of a spheric triangle are 60°, 100° and 80°, the area of the triangle is ... spheric degrees. 9.....

10 If an angle of a spheric triangle is 70°, the side opposite this angle in the polar triangle is ... degrees. 10.....

11 If the angle of a lune drawn on a sphere whose radius is 6 inches is 30°, the area of the lune is ... square inches. [Answer may be left in terms of π.] 11.....

Directions (questions 12–15) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

12 If two lines are parallel to the same plane, (*a*) they must be parallel to each other, (*b*) they can not intersect each other or (*c*) they may be skew (not in the same plane). 12.....

13 If a line is oblique to a plane, (*a*) it is perpendicular to one and only one line in the plane, (*b*) it is perpendicular to more than one line in the plane or (*c*) it is not perpendicular to any line in the plane. 13.....

[OVER]

14 The locus of the centers of all spheres tangent to two given intersecting planes is (*a*) a plane, (*b*) a pair of planes or (*c*) a pair of lines. 14.....

15 If two zones drawn on the same or equal spheres have altitudes of 4 inches and 1 inch respectively, their areas are in the ratio (*a*) 2:1, (*b*) 4:1 or (*c*) 16:1 15.....

Directions (questions 16–20) — Indicate whether each statement is true or false by writing the word *true* or *false* on the dotted line at the right.

16 All sections of a cylinder are parallelograms. 16.....

17 If a plane is passed parallel to the base of a pyramid, the area of the section thus formed is to the area of the base as its distance from the vertex is to the altitude. 17.....

18 The volume of a sphere is equal to the surface of the sphere multiplied by one third the radius. 18.....

19 A sphere can be inscribed in or circumscribed about any tetrahedron. 19.....

20 The sum of the sides of a convex spheric polygon must be greater than 180°. 20.....

See instructions for parts II and III on page 1.

Part II

Answer two questions from this part.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Given line a perpendicular to plane P and line b perpendicular to plane Q. Prove that if P and Q intersect, a and b can not be parallel. [10]

23 Prove that the sum of the angles of a spheric triangle is greater than 180° and less than 540°. [10]

*24 Represent by means of a drawing the locus of points equally distant from a given plane and a given point outside the plane. What is the name given to this locus? [8, 2]

Part III

Answer three questions from this part.

25 How many feet of iron piping can be made from 5170 cubic inches of iron if the outer diameter of the pipe is 8 inches and the thickness of the pipe is 1 inch? [Use $\pi = \frac{22}{7}$] [10]

26 The edges of a parallelepiped are 3 inches, 4 inches and 5 inches. An angle of the base is 30° and a lateral edge makes with its projection on the base an angle of 40°. Find, correct to the *nearest cubic inch*, the volume of the parallelepiped. [10]

27 Show that the volume of the sphere which is circumscribed about a cube is approximately 5.2 times the volume of the sphere inscribed in the cube. [10]

28 Each of the following statements is sometimes true and sometimes false. In *each* case give *one* condition under which it is true and *one* condition under which it is false.

a Four points determine a sphere. [2]

b Two angles having their sides parallel are equal. [2]

c The locus of points equally distant from two parallel planes and at a given distance from a given line is a circle. [2]

d If the faces of a polyhedron are regular polygons, the polyhedron is regular. [2]

e The plane passed through two diagonally opposite edges of a parallelepiped divides the parallelepiped into two congruent triangular prisms. [2]

*This question is based on one of the optional topics and may be used in either part II or part III.