The University of the State of New York

SOLID GEOMETRY

Thursday, January 25, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school...Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–11) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Any two lines parallel to the same ... are parallel to each other.   1.................

2 Two planes P and Q are perpendicular to each other and a line l, not in Q, is perpendicular to P. Line l is ... to plane Q.   2.................

3 If the plane angle of a dihedral angle contains 60°, a point 10 inches from each face of the dihedral angle is ... inches from the edge.   3.................

4 A plane is passed parallel to the base of a cone and 2 inches from the vertex. If the ratio of the area of the section so formed to the area of the base is 1:9, the altitude of the cone is ... inches.   4.................

5 If the lateral edge of a prism is 8 and the perimeter of the right section is 20, the lateral area of the prism is ....   5.................

6 If the slant height of a regular hexagonal pyramid is 8 and a base edge is 2, the lateral area of the pyramid is ....   6.................

7 The lateral area L of a frustum of a cone of revolution whose slant height is l and the radii of whose bases are r and r' is given by the formula $L =$ ....   7.................

8 If the volumes of two similar cylinders are in the ratio 27:125, their total surfaces are in the ratio ....   8.................

9 A lune whose angle is 10° contains ... spheric degrees.   9.................

10 Two points, A and B, are 25 inches apart. The locus of points 15 inches from A and 20 inches from B is a ....   10.................

11 Two symmetric spheric triangles on the same sphere or on equal spheres are ....   11.................

Directions (questions 12–18) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the dotted line at the right the word *always, sometimes* or *never*.

12 The sum of the sides of a convex spheric polygon is less than 360°.   12.................

13 A regular polyhedron may have a regular hexagon as its face.   13.................

14 The section formed by a plane intersecting the elements of a circular cylinder is a circle.   14.................

15 The sum of the angles of a spheric quadrilateral is greater than 360° and less than 720°.   15.................

16 If the radius of the base of a right circular cone whose altitude is h is increased by an amount x, the volume of the cone is increased by $\frac{1}{3}\pi x^2 h$.

16.....................

17 Two zones on the same or equal spheres are to each other as their altitudes.

17.....................

18 How many degrees are there in the angle formed by a diagonal of a cube and its projection on one of the faces? [Give answer correct to the *nearest degree.*]

18.....................

19 The locus of points equidistant from three points not in the same straight line is (*a*) a point, (*b*) a line or (*c*) a plane. Which is correct, *a*, *b* or *c*?

19.....................

20 If two spheric triangles on the same or equal spheres have equal perimeters, their polar triangles must (*a*) be mutually equiangular, (*b*) be mutually equilateral or (*c*) have equal areas. Which is correct, *a*, *b* or *c*?

20.....................

See instructions for groups II and III on page 1.

Group II
Answer three questions from this group.

21 Prove that a line perpendicular to one of two parallel planes is perpendicular to the other also. [10]

22 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle passing through these points. [10]

23 Given a rectangular parallelepiped and any two of its diagonals
Prove:

  a The two diagonals are equal   [5]

  b A sphere can be circumscribed about the parallelepiped   [5]

24 A sphere is inscribed in a right circular cylinder. Prove that the ratio of their total areas is equal to the ratio of their volumes. [10]

*25 The lower base edge of a frustum of a regular triangular pyramid is e_1, that of the upper base is e_2, and the altitude is h. Starting with the formula for the volume of a prismatoid,

$$V = \frac{h}{6} (B + B' + 4m),$$ show that the volume of the frustum is given by the formula

$$V = \frac{h \sqrt{3}}{12} (e_1{}^2 + e_2{}^2 + e_1 e_2). \quad [10]$$

Group III
Answer two questions from this group.

26 The accompanying figure represents a coal bin. The portion AB is rectangular, with base 12 feet × 12 feet and height 10 feet. AC is a frustum of a regular pyramid whose lower base edge is 2 feet and whose height is 15 feet. Find, correct to the *nearest ton*, the amount of coal necessary to fill the bin if one ton occupies 35 cubic feet of space.
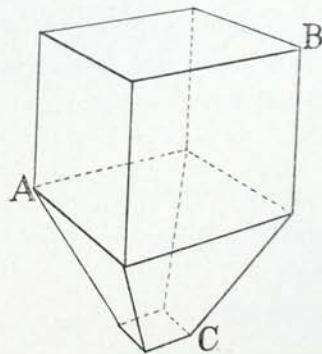
  [The formula for the volume of a frustum of

a pyramid is $V = \dfrac{h}{3} (B + B' + \sqrt{BB'})$]   [10]

27 A zone and an equilateral spheric triangle one of whose angles is 75° are drawn on the same sphere. If the zone and the triangle are equal, show that the ratio between the altitude of the zone and the radius of the sphere is 1 : 8 [10]

28 $A-BCD$ is a regular tetrahedron of edge e, and AO is the altitude to the base BCD. Points D and O are joined and the line is extended to meet BC at E. AE is then drawn.

  a Express AE in terms of e. [2]

  b Express EO in terms of e. [2]

  c Express AO in terms of e. [3]

  d Find angle AEO correct to the *nearest degree*. [3]

  * This question is based on one of the optional topics in the syllabus.

[2]