SOLID GEOMETRY

Tuesday, January 24, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.
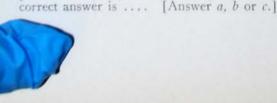
The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school..Name of pupil...

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–14) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The acute angle that a line makes with its projection on a plane is (a) greater than, (b) equal to or (c) less than, the angle it makes with any other line of the plane. The correct answer is .... [Answer a, b or c.]

1....................

2 Every section of a circular cylinder made by a plane containing an element is a ....

2....................

3 The lateral area S of a right circular cone, expressed as a function of its radius r and its altitude h, is $S = $ ....

3....................

4 The lateral areas of two similar cones are 16 and 25. If the altitude of the smaller cone is h, the altitude of the larger cone in terms of h is ....

4....................

5 The radius of a sphere whose area is 36π square inches is ... inches.

5....................

6 The area of a zone is equal to the product of its ... and the circumference of a great circle of the sphere on which it is drawn.

6....................

7 The area of a lune is to the area of the sphere on which it is drawn as the number of degrees in the angle of the lune is to ....

7....................

8 A ... perpendicular to the edge of a dihedral angle is perpendicular to each of its faces.

8....................

9 The ratio of the diagonal of a cube to the diagonal of one if its faces is .... [Answer may be left in radical form.]

9....................

10 There are ... regular polyhedrons.

10....................

11 The lower limit of the sum of the angles of a spheric triangle is ....

11....................

12 The upper limit of the sum of the face angles of any convex polyhedral angle is ....

12....................

13 The plane angle of a dihedral angle contains 68°. A point inside the angle, equidistant from each plane, is 5 inches from the edge of the angle. The distance of the point from either plane, correct to the *nearest tenth*, is ... inches.

13....................

14 The locus of points at a given distance from a given line is (a) two parallel lines, (b) two parallel planes or (c) a cylindric surface. The correct answer is .... [Answer a, b or c.]

14....................

Directions (questions 15–20) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the dotted line at the right the word *always, sometimes* or *never*.

15 Adjacent dihedral angles formed by two intersecting planes are equal.   15................

16 Parallel lines make equal angles with their projections on the same plane.   16................

17 The area of a spheric polygon measured in spheric degrees is greater than its spheric excess.   17................

18 If two parallel lines are each parallel to a plane, the plane of these lines is parallel to the given plane.   18................

19 The lateral faces of a frustum of a pyramid are trapezoids.   19................

20 The volume of a circular cylinder is doubled if its radius is doubled and its altitude is divided by 2.   20................

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Group II

Answer three questions from this group.

21 Prove that if the first of two spheric triangles is the polar triangle of the second, then the second is the polar triangle of the first.   [10]

22 Prove that every section of a circular cone made by a plane parallel to its base is a circle. [10]

23 Prove that if a line is perpendicular to one of two intersecting planes, its projection on the other plane is perpendicular to the line of intersection of the two planes.   [10]

24 Derive the formula for the total area T of a frustum of a regular quadrangular pyramid whose base edges are a and b and whose slant height is l.   [10]

25 M and N are two intersecting planes perpendicular to each other, and A is a point in their intersection.

   a What is the locus of points which are 3 inches from plane M?   [2]
   b What is the locus of points which are 4 inches from plane N?   [2]
   c What is the locus of points which are 6 inches from point A?   [2]
   d How many points are there which satisfy all three conditions stated in *a*, *b* and *c*?   [4]

Group III

Answer two questions from this group.

26 The area of a lune whose angle is 30° and that of a spheric triangle drawn on the same sphere are equal. The angles of the triangle are in the ratio 3:4:5. Find the number of degrees in each angle of the triangle.   [10]

27 A right triangle whose legs are 15 and 20 is revolved about the hypotenuse through an angle of 360°. Find the volume of the solid generated. [Use $\pi = 3.14$]   [10]

28 The volume of a sphere is equal to that of a right circular cylinder whose altitude is 8.75 and the radius of whose base is 3.22. Find, correct to the *nearest tenth*, the radius of the sphere.   [10]