The University of the State of New York

271ST HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 27, 1938 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–11) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The surface S of a sphere whose radius is r is expressed by the formula $S = \ldots$. 1.....

2 A convex polyhedral angle has *five* face angles. If all the face angles are equal, each angle lies between 0° and ...° and may have any value between these limits. 2.....

3 The diagonals of a rectangular parallelepiped meet in a point which is equidistant from the ... of the parallelepiped. 3.....

4 If the area of the surface of a sphere is 60 square inches, the area of a spheric triangle drawn on the surface of the sphere, whose angles are 80°, 100° and 72°, is ... square inches. 4.....

5 If the edge of a cube is 6 inches, the volume of the sphere inscribed in the cube is ... cubic inches. [Answer may be left in terms of π.] 5.....

6 If in a sphere whose radius is 13 inches the area of a section made by a plane is 25π square inches, the plane is ... inches from the center of the sphere. 6.....

7 If a line segment whose length is 8 is inclined 53° to a plane, the length of its projection on the plane, correct to the *nearest tenth*, is .... 7.....

8 A dihedral angle of 120° is bisected by a plane. A point P in this plane is 10 inches from the edge of the angle. The length of the perpendicular from P to either face of the angle is ... inches. [Answer may be left in radical form.] 8.....

9 The lateral area of a regular quadrangular pyramid circumscribed about a cone of revolution whose slant height is 5 inches and the radius of whose base is 4 inches is ... square inches. 9.....

10 If the areas of two similar solids are in the ratio 4:9, their volumes are in the ratio .... 10.....

11 If the radius of a sphere is 4 inches, a plane passed 1 inch from the center of the sphere divides the surface of the sphere in the ratio .... 11.....

Directions (questions 12–16) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter *a*, *b* or *c*.

12 The locus of the centers of equal small circles drawn on the surface of a given sphere is (*a*) the surface of the given sphere, (*b*) the surface of a sphere concentric with the given sphere or (*c*) a diameter of the sphere. 12.....

13 A regular octahedron has (*a*) 8 edges, (*b*) 12 edges or (*c*) 16 edges. 13.....

14 A section of a cone of revolution made by a plane containing an element is (*a*) an isosceles triangle, (*b*) an equilateral triangle or (*c*) a right triangle. 14.....

[OVER]

15 The area of the portion of the earth's surface which is included between the meridians 10° west and 50° west is (*a*) $\frac{1}{6}$ the surface of the earth, (*b*) $\frac{5}{9}$ the surface of the earth or (*c*) $\frac{1}{9}$ the surface of the earth. [Assume that the earth is a sphere.] 15.....

16 A straight line which is parallel to a plane is parallel to (*a*) only one line in the plane, (*b*) more than one line in the plane or (*c*) every line in the plane. 16.....

Directions (questions 17–20) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing on the dotted line at the right the word *always*, *sometimes* or *never*.

17 A plane is passed parallel to the base of a pyramid and bisecting the altitude. The area of the section made by the plane is one fourth of the area of the base. 17.....

18 The polar triangle of a spheric isosceles triangle is isosceles. 18.....

19 A plane drawn perpendicular to a lateral edge of a prism is parallel to the bases of the prism. 19.....

20 A diagonal of a cube makes an angle of 45° with each edge that it intersects. 20.....

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See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Prove that a spheric angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

23 Points D and E are midpoints of sides AC and BC respectively of triangle ABC. Prove that points A and B are equally distant from any plane passing through line DE. [10]

24 Planes P and Q are parallel. Line l which is perpendicular to plane P intersects P in point A and Q in point B.

a Describe the locus of the midpoints of all line segments included between P and Q. [3]

b Describe the locus of points which are equidistant from line l and plane P. [5]

c What geometric figure is formed by the intersection of these two loci? [2]

25 *a* Show how to construct a plane tangent to a given sphere and parallel to a given plane. [7]

b Prove that your method of construction is correct. [3]

Group III

Answer two questions from this group.

26 An isosceles trapezoid with bases 20 inches and 36 inches and legs 17 inches each is revolved about its longer base as an axis through an angle of 360°. Find the volume of the solid generated. [Answer may be left in terms of π.] [10]

27 A solid metal bar has the form of a right prism whose lateral edges are 11.0 inches and whose base is a right triangle with legs 8.0 inches and 7.0 inches. The bar is melted and recast into a hemisphere. Find, correct to the *nearest tenth of an inch*, the radius of the hemisphere. [Use $\pi = \frac{22}{7}$.] [10]

28 The altitude of a regular hexagonal pyramid is h and an edge of the base is a.

a Express the lateral area L of the pyramid in terms of a and h. [5]

b If $a = 10$ and $h = 14$, find, correct to the *nearest degree*, the dihedral angle formed by a lateral face and the base of the pyramid. [5]