The University of the State of New York

262D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, January 25, 1935 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

SOLID GEOMETRY

Friday, January 25, 1935

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Through any given point an unlimited number of ... can be passed perpendicular to a given plane. *Ans*.....

2 If a plane is passed parallel to the base of a pyramid and bisects the altitude of the pyramid, the area of the base of the pyramid is exactly ... times the area of the section formed by the cutting plane. *Ans*.....

3 At any given point on the surface of a sphere there can be only one ... which is tangent to the sphere. *Ans*.....

4 The distance from the center of a sphere whose radius is 15 to the plane of a small circle whose radius is 12 is .... *Ans*.....

5 If two intersecting lines a and b lying in plane P are each perpendicular to a third line c, then c is ... to P. *Ans*.....

6 The formula for the area A of a zone of height h on a sphere whose radius is r is $A =$ .... *Ans*.....

7 A plane perpendicular to the faces of a dihedral angle is ... to the edge of the dihedral angle. *Ans*.....

8 A line parallel to one of two perpendicular planes is perpendicular to the other. This statement is ... true. [Answer *always, sometimes* or *never.*] *Ans*.....

9 Of the five regular convex polyhedrons, the cube and the ... have the same number of edges. *Ans*.....

10 If the total surface of a cube is 24 square inches, the volume of the cube is ... cubic inches. *Ans*.....

11 If the volume of an oblique prism is 30 cubic inches and the lateral edge is 5 inches, the area of the right section of the prism is ... square inches. *Ans*.....

12 If the radius of the base of a right circular cone is r and the slant height is $2r$, the lateral area of the cone in terms of π and r is .... *Ans*.....

13 If the height of a right circular cone is multiplied by n and the radius of the base remains the same, the volume is multiplied by .... *Ans*.....

14 The locus of points within a given plane and at a given distance from a given line which is perpendicular to the plane is .... [Answer *a line, a circle* or *an ellipse.*] *Ans*.....

15 The number of spheric degrees in a lune whose angle is 1° is .... *Ans*.....

16 If the base edges of a frustum of a regular square pyramid are 4 and 10 inches respectively, and the altitude of the frustum is 4 inches, the slant height of the frustum is ... inches. *Ans*.....

17 If the volume of a sphere is $\frac{9\pi}{2}$, the surface of the sphere in terms of π is .... *Ans*.....

18 If the sides of a spheric triangle are 85°, 100° and 75°, the number of spheric degrees in the area of its polar triangle is .... *Ans*.....

19 Two points A and B are at distances 8 and 10 inches respectively from a given plane. If the line segment AB is 3 inches in length, the inclination of the line segment to the given plane, correct to the *nearest degree,* is .... *Ans*.....

20 In an equilateral spheric triangle each angle is greater than ... degrees and less than 180° and may have any value between these limits. *Ans*.....

SOLID GEOMETRY

Friday, January 25, 1935

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.
The minimum time requirement is five recitations a week for half a school year.
Name the author of the textbook you have used in solid geometry.

Part II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that if two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, they are equal and their planes are parallel. [10]

22 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

23 Prove that if line a in plane P is parallel to line b in plane Q and if P intersects Q in line c, then b and c are parallel. [10]

24 Given plane P and any three points, A, B and C, which are not in the same straight line and not in plane P

a Explain fully how you would locate a point in P equally distant from A, B and C. [8]

b Under what condition, if any, would there be more than one point satisfying the conditions given in *a*? [1]

c Under what condition, if any, would there be no point satisfying the conditions given in *a*? [1]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ *unless otherwise stated.*

25 A circle is inscribed in an equilateral triangle and the figure is revolved through 180° about an altitude of the triangle as an axis. Prove that the surface generated by the circle is two thirds of the lateral surface generated by the triangle. [10]

26 In a regular square pyramid the base edge is a and the slant height makes with the base an angle x.

a Derive a formula for the *total* surface T of the pyramid in terms of a and x. [6]

b Find, correct to the *nearest integer*, the value of T if $a = 2.2$ and $x = 47°$ [4]

27 A chimney 126 feet high has a flue in the shape of a right circular cylinder 3 feet in diameter. The top and the bottom of the chimney are circles whose diameters are 8 feet and 28 feet respectively. Find, correct to the *nearest cubic yard*, the number of cubic yards of masonry in the chimney. [The volume of the frustum of a right circular cone is given by the formula $V = \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$] [10]

192