

The University of the State of New York

259TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, January 26, 1934 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

- 1 Two . . . perpendicular to the same line are always parallel. Ans.....
- 2 In any convex polyhedron the sum of the face angles at any vertex is less than Ans.....
- 3 The ratio of the diagonal of a cube to the diagonal of one of its faces is Ans.....
- 4 A paper cone is made by rolling up a sector of a circle whose radius is 8 inches and whose central angle is 90° . The radius of the base of the cone is . . . inches. Ans.....
- 5 The altitude of a pyramid is 100 feet. Its base is a parallelogram whose sides are 6 feet and 8 feet and whose included angle is 45° . The volume of the pyramid is . . . cubic feet. [Answer may be left in radical form.] Ans.....
- 6 If the slant height of a cone of revolution is twice the radius of its base, the ratio of the lateral area to the area of the base is . . . Ans.....
- 7 The locus of points at a given distance from a given line of unlimited length is a . . . surface. Ans.....
- 8 A right prism has for its base an equilateral triangle of side a . If the altitude of the prism is h , the formula for the volume (V) in terms of a and h , is $V = \dots$ Ans.....
- 9 The altitude of a regular pyramid is 6 inches and its base is a square whose side is 9 inches. The area of a section made by a plane parallel to the base and at a distance of 2 inches from the base is . . . square inches. Ans.....
- 10 The volumes of two cubes are 64 and 27; the ratio of their surfaces is Ans.....
- 11 The area in spherical degrees of a spheric triangle, one of whose angles is a right angle, is 30; if one of the other angles of the triangle is 50° , the third angle contains . . . degrees. Ans.....
- 12 The locus of all points within a trihedral angle and equidistant from the faces is a Ans.....

13 A diagonal of a rectangular parallelepiped is 12 inches long and makes an angle of 34° with the base; the length of a diagonal of the base correct to the nearest tenth is . . . inches.

Ans.

14 If the number of square inches in the surface of a sphere equals the number of inches in the circumference of a great circle, the volume of the sphere is . . . cubic inches. [Answer may be left in terms of π .]

Ans.

15 On a sphere whose radius is 9 inches is a lune whose angle is 40° . The area of the lune is . . . square inches. [Answer may be left in terms of π .]

Ans.

Directions (questions 16-20) — Some of these statements are *always* true, others are *sometimes* true and the rest are *never* true. Indicate your opinion in each case by writing the word *always*, *sometimes* or *never* on the dotted line at the right.

16 If two lines are parallel, a plane containing one is parallel to a plane containing the other.

Ans.

17 If two planes intersect, any two adjacent dihedral angles are equal.

Ans.

18 The face angles at the vertex of a regular pyramid are equal.

Ans.

19 The volume of a circular cylinder is doubled if its altitude is doubled and its radius halved.

Ans.

20 Any line which meets a plane obliquely meets one line in the plane at right angles.

Ans.

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Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

Name the author of the textbook you have used in solid geometry.

Part III

Answer five questions from part III, including three questions from group I and two questions from group II.

Group I

Answer five questions from this group.

- 21 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]
- 22 Prove that if a pyramid is cut by a plane parallel to its base, the section is a polygon similar to the base. [10]
- 23 Prove that if two parallel lines are oblique to a plane, their projections on that plane are parallel. [10]
- 24 Prove that the section formed by a plane passing through a tetrahedron parallel to two opposite edges is a parallelogram. [10]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.

- 25 The sides of a spheric triangle on a sphere of radius 15 inches are 90° , 63° , 46° ; find the number of square inches in the area of the polar triangle. [Leave answer in terms of π .] [10]
- 26 Cleopatra's Needle, an Egyptian obelisk in Central Park, New York City, has the form of a frustum of a quadrangular pyramid surmounted by a pyramid. The frustum is 8 feet square at the base, 5 feet square at the top and 64 feet high. The pyramid that surmounts the frustum is 5 feet square at the base and 7 feet high. Assuming 165 pounds per cubic foot, find to the nearest ton the weight of the obelisk. [The formula for the volume of a frustum of a pyramid is $V = \frac{h}{3}(B + B' + \sqrt{BB'})$] [10]

27 The altitude of a right circular cone is h and the angle formed by an element and the base is A .

- a Derive a formula for the volume (V) of a cone in terms of h and A . [5]
- b From the formula derived in answer to a find, by the use of logarithms, the volume of the cone correct to the nearest tenth of a cubic foot if $h = 21$ feet and angle $A = 45^\circ$ [Use $\pi = 3.14$] [5]