The University of the State of New York

256TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, January 27, 1933 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–8) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The number of diagonals of a cube is .... *Ans*.....

2 A sphere can be circumscribed about any ... parallelepiped. *Ans*.....

3 The radius of the earth is approximately 4000 miles; hence Leningrad, which is on the 60th parallel of latitude (north) is ... miles from the north pole. *Ans*.....

4 If one side of the base of a regular hexagonal pyramid is 4 inches and a lateral edge is 5 inches, the volume is ... cubic inches. *Ans*.....

5 The locus of the vertices of all right circular cones having a fixed circle as base is a .... *Ans*.....

6 If the angles of a spheric triangle are 63°, 101° and 88°, the ratio of the area of the triangle to that of the sphere is .... *Ans*.....

7 If the slant height of a regular pyramid with a square base is 25 and the perimeter of the base is 56, then the altitude of the pyramid is .... *Ans*.....

8 If a solid sphere of metal with a diameter of 6 inches weighs 27 pounds, a hollow spherical shell of the same metal one inch thick and with the same outer diameter will weigh ... pounds. *Ans*.....

Directions (questions 9–15) — Indicate in the space at the right of each statement whether it is true or false.

9 The plane passed through two diagonally opposite edges of any parallelepiped divides it into two congruent triangular prisms. *Ans*.....

10 Two lines perpendicular to the same line are always parallel. *Ans*.....

11 Through a given point outside a given plane one and only one plane can be passed perpendicular to the given plane. *Ans*.....

12 The lateral area of any prism is equal to the product of the perimeter of the base and a lateral edge. *Ans*.....

13 A diagonal of a cube makes an angle of 45° with each of the three edges which it meets at a vertex. *Ans*.....

14 Any two diagonals of a cube are perpendicular to each other. *Ans*.....

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15 Two planes perpendicular to the same plane are always parallel to each other. *Ans*.....

Directions (questions 16–20) — Indicate your answer in the space at the right of each statement by the correct letter.

16 The locus of the centers of all spheres tangent to two given intersecting lines is (*a*)a plane (*b*)two planes (*c*)two lines (*d*)a sphere *Ans*.....

17 A fixed line l is perpendicular to a fixed plane MN and a point P moves so that it is always just as far from l as from MN. The locus of P is (*a*)a circle (*b*)a sphere (*c*)a plane (*d*)a conical surface *Ans*.....

18 If in computing the volume of a sphere $\frac{22}{7}$ is used as the value of π, the result will be (*a*)greater than (*b*)less than the actual volume. *Ans*.....

19 A sphere is inscribed in a cube. The part of the volume of the cube outside the sphere is (*a*)greater than (*b*)less than (*c*)equal to the volume inside the sphere. *Ans*.....

20 If the earth is assumed to be a sphere, the area of the part of the northern hemisphere south of the 30° parallel of latitude is (*a*)greater than (*b*)less than (*c*)equal to the area of the part north of this parallel. *Ans*.....

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Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

Name the author of the textbook you have used in solid geometry.

Part II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that the sum of any two face angles of a trihedral angle is greater than the third face angle. [10]

22 Prove that every section of a circular cone made by a plane parallel to the base is a circle. [10]

23 If AB and CD are any two chords of a sphere intersecting at an interior point P, prove $PA \times PB = PC \times PD$ [10]

24 If A–BCD is a regular tetrahedron, and P and Q are the centers of the faces BCD and ACD respectively, prove that PQ is parallel to AB and equal to one third of AB. [10]

25 Prove that the plane passing through the center of a cube and perpendicular to one of its diagonals cuts the cube in a regular hexagon. [10]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.

26 A spheric triangle has an area of 44 square inches and angles of 70°, 75° and 105°. What is the radius of the sphere? [10]

27 A slice is cut from a sphere whose radius is 5 inches by a plane 4 inches from the center. The slice has one plane face and one convex face. Show that the area of the convex face exceeds the area of the plane face by exactly π square inches. [10]

28 A regular pyramid has a square base and its lateral faces are equilateral triangles. Find the angle that the altitude makes with one of the lateral edges. [10]