# SOLID GEOMETRY

Friday, January 23, 1931 — 9.15 a. m. to 12.15 p. m., only

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### Instructions

*Do not open this sheet until the signal is given.*

*Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.*

*Part I is to be done first and the maximum time to be allowed for this part is one hour.* Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

# SOLID GEOMETRY

## Friday, January 23, 1931

### Fill in the following lines:

Name of school................................................Name of pupil................................................

**Detach this sheet and hand it in at the close of the one hour period.**

### Part I

*Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.*

1 How many diagonals has every parallelepiped?

Ans...............

2 If a straight line is parallel to a plane, is it parallel to every line in the plane?  [Answer *yes* or *no*.]

Ans...............

3 If two angles have their sides respectively parallel, are the angles necessarily equal?  [Answer *yes* or *no*.]

Ans...............

4 The plane of an acute angle is rotated about one of the sides of the angle as an axis; what kind of surface is generated by the other side?

Ans...............

5 Does a straight line segment moving through space always generate a surface?  [Answer *yes* or *no*.]

Ans...............

6 Can the same straight line lie in two different surfaces that are not both planes?  [Answer *yes* or *no*.]

Ans...............

7 What is the least number of rectangular faces a right parallelepiped can have?

Ans...............

8 If a diagonal of one cube is an edge of a second cube, what is the ratio of the volumes of the two cubes?  [Answer may be left in radical form.]

Ans...............

9 What is the lateral area of a regular hexagonal pyramid whose base edge is 6 and whose altitude is 8?  [Answer may be left in radical form.]

Ans...............

10 The altitude of a pyramid is 6 inches.  A section parallel to the base is 5 inches from the base.  What is the ratio of the section to the base?

Ans...............

11 Write the formula for the lateral area $S$ of a right circular cone of radius $r$ and slant height $l$.

Ans...............

12 Find the volume of a right circular cone whose altitude is 6 inches and whose elements are inclined 60° to the base.  [Answer may be left in terms of $\pi$.]

Ans...............

13 What is the radius of the sphere whose volume is numerically equal to its area?

Ans...............

14 A right circular cylinder and a sphere have equal radii.  If the lateral area of the cylinder equals the area of the sphere, the altitude of the cylinder is how many times the radius of the sphere?

Ans...............

15 By what number must the edge of a cube be multiplied to double the surface?

Ans...............

16 What is the spheric excess of a spheric polygon whose four angles are 80°, 95°, 140° and 101°?

Ans...............

17 What is the relation between two planes if a circle in one of them projects into a straight-line segment in the other?                     *Ans.*............

18 What is the area in square inches of a lune of 54° on a sphere of radius 7 inches?   [Answer may be left in terms of $\pi$.]                     *Ans.*............

19 Small circles of a given radius are drawn on a given sphere; what geometric figure is the locus of the centers of all such circles?                     *Ans.*............

20 What figure is formed by the intersection of two spheres?                     *Ans.*............

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## Friday, January 23, 1931

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

Name the author of the textbook you have used in solid geometry.

### Part II

*Answer five questions from part II, including three questions from group I and two questions from group II.*

### Group I

#### Answer three questions from this group.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [12]

22 Prove that if the first of two spheric triangles is the polar triangle of the second, then the second is the polar triangle of the first. [12]

23 Prove that two parallel line segments that are oblique to a plane have the same ratio as their projections on the plane. [12]

24 Describe fully the complete locus of points equidistant from two intersecting planes and at a given distance from their line of intersection. [4, 4, 4]

### Group II

#### Answer two questions from this group.

*Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.*

25 A vertical drainpipe in the shape of a cylinder 4 inches in diameter and 30 feet long extends from the eaves of a house to the ground. It is stopped up 6 feet from the ground and is filled with water from this point to the top. Calculate to the *nearest pound* the weight of water resting in the pipe. [1 cubic foot of water weighs 62.5 pounds.] [12]

26 Find to the *nearest tenth of a quart* the capacity of a pail shaped like a frustum of a right circular cone whose upper radius is 6 inches, lower radius 5 inches and slant height 9 inches. $[V = \frac{1}{3}h (B + B' + \sqrt{BB'})$; 1 gallon = 231 cubic inches] [12]

27 The hypotenuse of a right triangle is 12 inches and one leg is 8 inches; find the volume of the solid generated by revolving the triangle about the hypotenuse as an axis. [Answer may be left in terms of $\pi$.] [12]