The University of the State of New York

235th High School Examination

SOLID GEOMETRY

Friday, January 22, 1926 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

1 Prove that if one of two parallel lines is perpendicular to a plane, the other line is also perpendicular to this plane. [12½]

2 Prove that if a straight line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [12½]

3 Prove that the section of a pyramid made by a plane parallel to the base is a polygon similar to the base. [12½]

4 Prove that the volume of any pyramid is equal to one third the product of its base and its altitude. [12½]

5 Prove that in two polar triangles each angle of one is measured by the supplement of the side lying opposite to it in the other. [12½]

Group II

Irrational results may be left in the form of π and radicals unless otherwise stated.

6 The diagonal of a cube is $7\sqrt{3}$; find its volume and its total surface. [6, 6½]

7 Prove that if a line and a plane are parallel, a second plane perpendicular to the line is also perpendicular to the given plane. [12½]

8 An oblique cone with a circular base has an altitude of 8 feet. Its shortest and longest elements are 10 feet and 17 feet respectively. Find the volume of the cone. [12½]

9 a If the radius of a sphere is increased 50%, by what per cent is its volume increased? By what per cent is its surface increased? [3, 3]

b Show that the area of a trirectangular spheric triangle on the surface of a sphere is equal to one eighth of the area of the sphere. [6½]

10 Given a sphere whose radius is 2" and a fixed line through its center; find the locus of points in space 3" from the surface of the sphere and 3" from the line. [5, 5, 2½]

11 A watering trough in the form of a regular triangular prism has one of its lateral edges on the ground and the opposite face horizontal. It is filled with water to a depth of two thirds of the altitude of the trough in this position. What part of the volume of the trough is occupied by the water? [12½]

12 State whether *each* of the following statements is true or false: [Label each answer with the corresponding letter.]

a If a right circular cylinder circumscribes a sphere, the lateral area of the cylinder is greater than the surface of the sphere. [2½]

b Three planes tangent to the same sphere can not be parallel. [2]

c Three planes parallel to the same line are parallel to each other. [2]

d Three planes may have only one point in common. [2]

e Two faces of a truncated prism may be parallel. [2]

f Any face angle of a trihedral angle is greater than the difference between the other two face angles. [2]