The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

# COURSE III

Thursday, August 14, 1997 — 8:30 to 11:30 a.m., only

**Notice . . .**

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

# Formulas

## Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

## Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

## Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

## Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

## Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

## Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

# Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of $\pi$ or in radical form. [60]

1 If $f(x) = \sqrt{3x} + \sqrt{12x}$, express $f(-3)$ as a monomial in terms of $i$.

2 If $7^{(x^2+x)} = 49$, find the positive value of $x$.

3 Express $160°$ in radian measure.

4 If $\cos 72° = \sin x$, find the number of degrees in the measure of acute angle $x$.

5 If $f(x) = x^{\frac{1}{2}} + x^{-2}$, what is the value of $f(4)$?

6 Find the value of $\displaystyle\sum_{x=0}^{2} 2^x$.

7 Perform the indicated operations and express in simplest form:

$$\frac{a+8}{7a^2} \cdot \frac{3a^2 - 24a}{a^2 - 64}$$

8 Express $\sin 75° \cos 15° - \cos 75° \sin 15°$ as a single trigonometric function of a positive acute angle.

9 If $f(x) = x^2$ and $g(x) = x + 1$, what is $(f \circ g)(2)$?

10 In which quadrant does the graph of the sum of $6 + 4i$ and $3 - 5i$ lie?

11 Two tangents are drawn to circle $O$ from external point $P$. If the major arc formed has a measure of $280°$, find $m\angle P$.

12 If $\sin \theta = -\frac{8}{17}$ and $\tan \theta$ is positive, what is the value of $\cos \theta$?

13 Find, to the *nearest tenth*, the positive value of $x$ in the equation $\sqrt{x^2 + 21} = 2x$.

14 If the coordinates of $A$ are $(2,-3)$, what are the coordinates of $A'$, the image of $A$ after $R_{90°} \circ r_{y\text{-axis}} (A)$?

15 In the accompanying diagram, $\overline{AD}$ is tangent to circle $O$ at $D$ and $\overline{ABC}$ is a secant. If $AD = 6$ and $AC = 9$, find $AB$.



*Directions* (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 In the accompanying table, $y$ varies inversely as $x$.

| $x$ | 3 | 6 | 12 |
|-----|---|---|----|
| $y$ | 8 | 4 | $z$ |

What is the value of $z$?

(1) $\frac{1}{2}$      (3) 3

(2) 2      (4) $\frac{1}{4}$

17 What is the solution set of the equation $|2x + 1| = 9$?

(1) $\{-5\}$  (3) $\{4,-5\}$
(2) $\{-4,5\}$  (4) $\{4\}$

18 The expression $\log\left(\dfrac{x^n}{\sqrt{y}}\right)$ is equivalent to

(1) $n \log x - \frac{1}{2} \log y$
(2) $n \log x - 2 \log y$
(3) $\log (nx) - \log \left(\frac{1}{2}y\right)$
(4) $\log (nx) - \log (2y)$

19 Which value is *not* in the range of the equation $y = \sin x$?

(1) $1$  (3) $\frac{1}{2}$
(2) $2$  (4) $-\frac{1}{2}$

20 Which trigonometric function is equivalent to the expression $\dfrac{\sin 2x}{2 \sin x}$?

(1) $\tan x$  (3) $\sin x$
(2) $\cot x$  (4) $\cos x$

21 For which value of $\theta$ is the expression $\dfrac{2}{\tan \theta - 1}$ undefined?

(1) $0$  (3) $\frac{\pi}{4}$
(2) $\frac{3\pi}{4}$  (4) $-\frac{\pi}{4}$

22 The value of $\sin\left(\dfrac{3\pi}{2}\right) - \cos\left(\dfrac{\pi}{3}\right)$ is

(1) $-1\frac{1}{2}$  (3) $\frac{1}{2}$
(2) $1\frac{1}{2}$  (4) $-\frac{1}{2}$

23 In which quadrants does the equation $xy = 10$ lie?

(1) I and II  (3) II and IV
(2) I and III  (4) III and IV

24 A set of scores with a normal distribution has a mean of 32 and a standard deviation of 3.7. Which score could be expected to occur the *least* often?

(1) 26  (3) 36
(2) 29  (4) 40

25 Which equation is represented in the graph below?
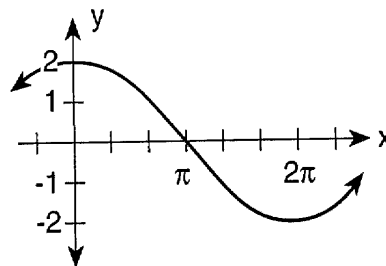


(1) $y = 2 \cos 2x$  (3) $y = 2 \cos \frac{1}{2}x$
(2) $y = \frac{1}{2} \cos 2x$  (4) $y = \frac{1}{2} \cos \frac{1}{2}x$

26 The product of $5 - 2i$ and $i$ is

(1) $7$  (3) $5 - 2i$
(2) $2 + 5i$  (4) $-2 + 5i$

27 Expressed in simplest form, $\dfrac{n - \dfrac{1}{n}}{1 + \dfrac{1}{n}}$ is equivalent to

(1) $n - 1$  (3) $\dfrac{n - 1}{n + 1}$
(2) $n + 1$  (4) $n$

28 Which is an equation of the inverse of $y = \frac{3}{2}x$?

(1) $y = \frac{2}{3}x$  (3) $y = 3x - 2$
(2) $y = -\frac{3}{2}x$  (4) $y = \dfrac{x + 3}{2}$

29 In a triangle, the sides measure 3, 5, and 7. What is the measure, in degrees, of the largest angle?

(1) 60  (3) 120
(2) 90  (4) 150

30 In $\triangle ABC$, $m\angle A = 45$, $m\angle B = 30$, and side $a = 10$. What is the length of side $b$?

(1) $5\sqrt{2}$  (3) $10\sqrt{2}$

(2) $5\sqrt{3}$  (4) $10\sqrt{3}$

31 Which relation is *not* a function?

(1) $\{(x,y) \mid y = \cos x\}$  (3) $\{(x,y) \mid y = 3^x\}$
(2) $\{(x,y) \mid x = y\}$  (4) $\{(x,y) \mid x = 3\}$

32 The roots of the quadratic equation $4x^2 = 2 + 7x$ are best described as

(1) real, equal, and rational
(2) real, unequal, and rational
(3) real, unequal, and irrational
(4) imaginary

33 What is the area of a parallelogram if two adjacent sides measure 4 and 5 and an included angle measures 60°?

(1) $5\sqrt{2}$  (3) $5\sqrt{3}$

(2) $10\sqrt{2}$  (4) $10\sqrt{3}$

34 Which quadratic equation has roots $2 + i$ and $2 - i$?

(1) $x^2 + 4x + 5 = 0$  (3) $x^2 + 4x - 5 = 0$
(2) $x^2 - 4x - 5 = 0$  (4) $x^2 - 4x + 5 = 0$

35 If three fair coins are tossed, what is the probability of getting *at least* two heads?

(1) $\frac{1}{8}$  (3) $\frac{1}{2}$

(2) $\frac{3}{8}$  (4) $\frac{2}{3}$

**Answers to the following questions are to be written on paper provided by the school.**

## Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.    [40]

36 *a* Given the equation:   $y = 2 \sin \frac{1}{2}x$

(1) On graph paper, sketch and label the graph of this equation in the interval $0 \le x \le 2\pi$.    [4]

(2) On the same set of axes, sketch the image of the graph drawn in part *a*(1) after the transformation $r_{x\text{-axis}}$. Label the graph $T$.    [2]

(3) Write the equation of the graph drawn in part *a*(2).    [2]

*b* The graph below *incorrectly* represents the equation $y = 2 \cos x$. Write a mathematical explanation of why this graph is incorrect.    [2]
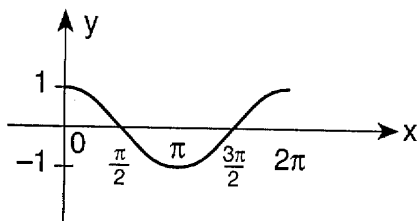


37 *a* On graph paper, sketch the graph of the equation $y = 2^x$ in the interval $-3 \le x \le 3$.    [2]

*b* On the same set of axes, reflect the graph drawn in part *a* in the $y$-axis and label it *b*.    [2]

*c* Write an equation of the function graphed in part *b*.    [2]

*d* On the same set of axes, reflect the graph drawn in part *a* in the line $y = x$ and label the reflection *d*.    [2]

*e* Write an equation of the function graphed in part *d*.    [2]
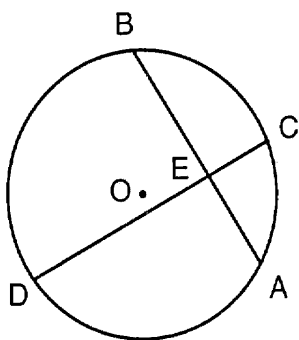
**38** *a* In the accompanying diagram, regular pentagon *ABCDE* is inscribed in circle *O*, chords $\overline{EC}$ and $\overline{DB}$ intersect at *F*, chord $\overline{DB}$ is extended to *G*, and tangent $\overline{GA}$ is drawn.
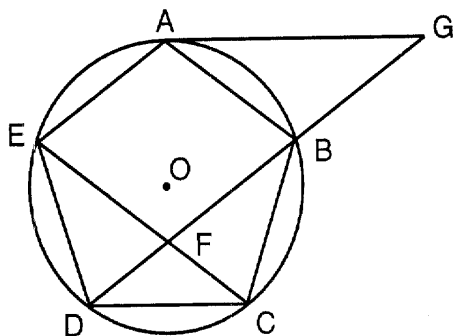


Find:

(1) m∠*BDE*　　[2]
(2) m∠*BFC*　　[2]
(3) m∠*AGD*　　[2]

*b* In the accompanying diagram of circle *O*, chords $\overline{AB}$ and $\overline{CD}$ intersect at *E*, *AE* = *x*, *EB* = *x* + 1, *CE* = *x* − 1, and *ED* = 2*x*. Find *AE*.　　[4]



**39** *a* Solve for *x* to the *nearest hundredth*:

$$6^x = 45 \qquad [4]$$

*b* Find, to the *nearest degree*, all values of *x* in the interval 0° ≤ *x* < 360° that satisfy the equation $3 \sin^2 x - 2 \sin x = 1$.　　[6]

**40** Two forces act on an object. The first force has a magnitude of 85 pounds and makes an angle of 31°30' with the resultant. The magnitude of the resultant is 130 pounds.

*a* Find the magnitude of the second force to the *nearest tenth of a pound*.　　[6]

*b* Using the results from part *a*, find, to the *nearest ten minutes* or *nearest tenth of a degree*, the angle that the second force makes with the resultant.　　[4]

**41** *a* If a letter is selected at random from the name MARILYN in five separate trials, what is the probability that the M is chosen *exactly* three times?　　[2]

*b* If a letter is selected at random from the name DAPHNE in seven separate trials, what is the probability that a vowel is chosen *at least* six times?　　[4]

*c* If a letter is selected at random from the name NORMA in six separate trials, what is the probability that a consonant is chosen *at most* once?　　[4]

**42** *a* For all values of *x* for which the expressions are defined, prove that the following is an identity:

$$\frac{\sin (A + B) + \sin (A - B)}{\sin (A + B) - \sin (A - B)} = \frac{\tan A}{\tan B} \qquad [6]$$

*b* During a 10-game season, a high school football team scored the following number of points:

　　14, 17, 21, 10, 35, 27, 13, 7, 45, 21

Find the standard deviation of these scores to the *nearest thousandth*.　　[4]

## The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# SEQUENTIAL MATH – COURSE III

**Thursday,** August 14, 1997 — 8:30 to 11:30 a.m., only

| | |
|---|---|
| Part I Score | . . . . . . . . . . . |
| Part II Score | . . . . . . . . . . . |
| Total Score | . . . . . . . . . . |
| Rater's Initials: | . . . . . . . . . . . . . |

## ANSWER SHEET

Pupil . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .   Sex: ☐ Male ☐ Female    Grade . . . . . . . . . .

Teacher . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .   School . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

**Your answers to Part I should be recorded on this answer sheet.**

### Part I

**Answer 30 questions from this part.**

| | | | |
|---|---|---|---|
| 1 . . . . . . . . . . . . . . . . . | 11 . . . . . . . . . . . . . . . . | 21 . . . . . . . . . . . . . . . . . | 31 . . . . . . . . . . . . . . . . . |
| 2 . . . . . . . . . . . . . . . . . | 12 . . . . . . . . . . . . . . . . | 22 . . . . . . . . . . . . . . . . | 32 . . . . . . . . . . . . . . . . . |
| 3 . . . . . . . . . . . . . . . . . | 13 . . . . . . . . . . . . . . . . | 23 . . . . . . . . . . . . . . . . | 33 . . . . . . . . . . . . . . . . . |
| 4 . . . . . . . . . . . . . . . . | 14 . . . . . . . . . . . . . . . . | 24 . . . . . . . . . . . . . . . . | 34 . . . . . . . . . . . . . . . . . |
| 5 . . . . . . . . . . . . . . . . | 15 . . . . . . . . . . . . . . . . | 25 . . . . . . . . . . . . . . . . | 35 . . . . . . . . . . . . . . . . . |
| 6 . . . . . . . . . . . . . . . . | 16 . . . . . . . . . . . . . . . . | 26 . . . . . . . . . . . . . . . . | |
| 7 . . . . . . . . . . . . . . . . | 17 . . . . . . . . . . . . . . . . | 27 . . . . . . . . . . . . . . . . | |
| 8 . . . . . . . . . . . . . . . . | 18 . . . . . . . . . . . . . . . . | 28 . . . . . . . . . . . . . . . . | |
| 9 . . . . . . . . . . . . . . . . | 19 . . . . . . . . . . . . . . . . | 29 . . . . . . . . . . . . . . . . | |
| 10 . . . . . . . . . . . . . . . . | 20 . . . . . . . . . . . . . . . . | 30 . . . . . . . . . . . . . . . . | |

**Your answers for Part II should be placed on paper provided by the school.**

**The declaration below should be signed when you have completed the examination.**

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

# FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

## THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

# COURSE III

**Thursday,** August 14, 1997 — 8:30 to 11:30 a.m., only

# SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

| | | | |
|---|---|---|---|
| (1) $9i$ | (11) 100 | (21) 3 | (31) 4 |
| (2) 1 | (12) $-\frac{15}{17}$ | (22) 1 | (32) 2 |
| (3) $\frac{8\pi}{9}$ | (13) 2.6 | (23) 2 | (33) 4 |
| (4) 18 | (14) (3,–2) | (24) 4 | (34) 4 |
| (5) $2\frac{1}{16}$ | (15) 4 | (25) 3 | (35) 3 |
| (6) 7 | (16) 2 | (26) 2 | |
| (7) $\frac{3}{7a}$ | (17) 3 | (27) 1 | |
| (8) $\sin 60°$ | (18) 1 | (28) 1 | |
| (9) 9 | (19) 2 | (29) 3 | |
| (10) IV | (20) 4 | (30) 1 | |

## Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* (3) $y = -2 \sin \frac{1}{2}x$    [2]

     *b* Amplitude should be 2.

             *or*    [2]

    an equivalent statement

(37) *c* $y = 2^{-x}$    [2]

     *e* $x = 2^y$ *or* $y = \log_2 x$    [2]

(38) *a* (1) 72    [2]

      (2) 72    [2]

      (3) 36    [2]

    *b* 3    [4]

(39) *a* 2.12    [4]

    *b* 90, 199, 341    [6]

(40) *a* 72.7    [6]

    *b* 37.7° *or* 37°40'    [4]

(41) *a* $\frac{360}{16807}$    [2]

    *b* $\frac{15}{2187}$    [4]

    *c* $\frac{640}{15625}$    [4]

(42) *b* 11.198    [4]