The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, August 13, 1992 — 8:30 to 11:30 a.m., only

Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet. Open the booklet and carefully remove the reference tables.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 What is the amplitude of the graph of the equation $y = 6 \cos 3x$?

2 Express 120° in radian measure.

3 Find the numerical value of the expression $\sin 30° + \cos 60°$.

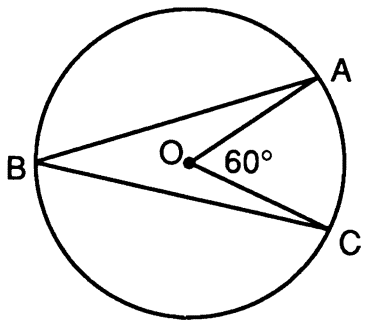
4 In $\triangle ABC$, $a = 10$, $b = 6$, and $\sin B = \frac{2}{5}$. Express $\sin A$ in simplest fractional form.

5 Write $(R_{-30} \circ R_{40} \circ R_{60})(A)$ as an equivalent single rotation of A.

6 Find the value of $\tan\left(\text{Arc}\cos\frac{\sqrt{2}}{2}\right)$.

7 Factor completely: $3x^3y^2 - 147x$

8 In the accompanying figure, central angle AOC measures 60°. What is the number of degrees in the measure of inscribed angle ABC?



9 Solve for x: $2^x = 4^{\frac{3}{2}}$

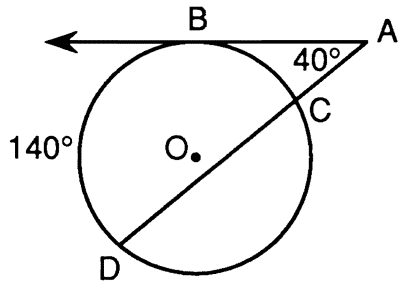
10 Solve for k: $\frac{3}{k} + 1 = \frac{1}{2}$

11 Solve for all values of x: $|4x - 1| = 3$

12 When a biased coin is tossed, the probability of getting a tail is $\frac{1}{4}$. If the coin is tossed two times, what is the probability of getting two heads?

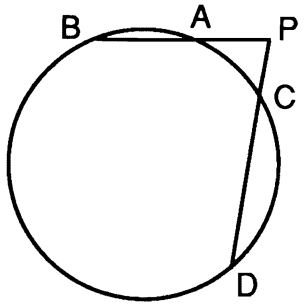
13 When represented graphically, in which quadrant does the sum of $-4 - i$ and $3 + 4i$ lie?

14 In the accompanying diagram, $\overleftarrow{AB}$ is tangent to circle O at B and $\overline{ACD}$ is a secant. If $m\angle A = 40$ and $m\overset{\frown}{BD} = 140$, find $m\overset{\frown}{BC}$.



15 Express in simplest form: $\dfrac{5 - \frac{25}{x}}{x - 5}$

16 In the diagram below, $\overline{PAB}$ and $\overline{PCD}$ are secants to the circle. If $PA = 4$, $AB = 5$, and $PD = 12$, what is PC?



Directions (17–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

17 If $f(x) = \frac{x + 3}{x - 1}$, then $f(a + 1)$ is equal to

(1) $\frac{a + 4}{a}$ (3) 5

(2) $\frac{a + 3}{a - 1}$ (4) 4

18 The expression $\frac{1}{3 + \sqrt{5}}$ is equivalent to

(1) $\frac{3 - \sqrt{5}}{4}$ (3) $\frac{3 + \sqrt{5}}{13}$

(2) $\frac{\sqrt{5}}{8}$ (4) $\frac{3 - \sqrt{5}}{8}$

19 Since $\sin 75° = \sin (30° + 45°)$, then $\sin 75°$ equals

(1) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (3) $\frac{-\sqrt{2} - \sqrt{6}}{4}$

(2) $\frac{-\sqrt{6} + \sqrt{2}}{4}$ (4) $\frac{\sqrt{2} + \sqrt{6}}{4}$

20 If $\tan A > 0$ and $\cos A < 0$, in which quadrant does $\angle A$ terminate?

(1) I (3) III

(2) II (4) IV

21 In $\triangle ABC$, $a = 6$, $b = 5$, and $c = 8$. Cos A equals

(1) $\frac{75}{80}$ (3) $-\frac{3}{80}$

(2) $\frac{53}{80}$ (4) $\frac{53}{60}$

22 What is the solution set of $x^2 - 8x + 12 < 0$?

(1) $\{x|x < 2 \vee x > 6\}$

(2) $\{x|x < -6 \vee x > -2\}$

(3) $\{x|2 < x < 6\}$

(4) $\{x|-6 < x < -2\}$

23 Given the real valued function $f(x) = \frac{1}{\sqrt{x - 3}}$, which number is in the domain of $f(x)$?

(1) -1 (3) 3

(2) 2 (4) 7

24 If $\sin \theta = \frac{2}{3}$ and θ is in Quadrant I, what is the value of $(\tan \theta)(\cos \theta)$?

(1) $\frac{2}{3}$ (3) $\frac{3\sqrt{5}}{5}$

(2) $\frac{\sqrt{5}}{3}$ (4) $\frac{2\sqrt{5}}{3}$

25 The test scores of 50 students resulted in a mean of 82 and a standard deviation of 7.5. If the distribution of scores was normal, which score could be expected to occur less than 5% of the time?

(1) 98 (3) 74

(2) 90 (4) 70

26 Log $\frac{\sqrt{b}}{a^2}$ is equivalent to

(1) $\frac{1}{2} \log b + 2 \log a$ (3) $2 \log b - \frac{1}{2} \log a$

(2) $\frac{1}{2} \log b - 2 \log a$ (4) $\frac{\frac{1}{2} \log b}{2 \log a}$

27 The expression $\frac{\cos^2 x + \sin^2 x}{\sin x}$ is equivalent to

(1) $\sin x$ (3) $\sec x$

(2) $\cos x$ (4) $\csc x$

28 What is the value of $\sum_{r=2}^{4} {}_6C_r$?

(1) 15 (3) 50

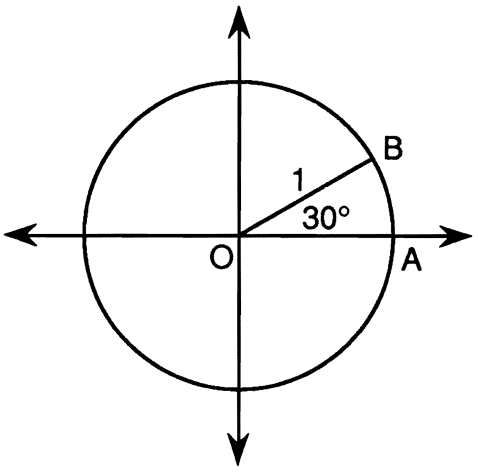
(2) 35 (4) 64

29 The value of $10^{1.9047}$ is

(1) 0.0803 (3) 8.03

(2) 0.803 (4) 80.3

30 In the accompanying diagram, the center of circle O is at the origin, radius $OB = 1$, and $m\angle AOB = 30$.



What are the coordinates of point B?

(1) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ (3) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

(2) $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ (4) $(1,1)$

31 If $\sqrt{-28}$ is subtracted from $\sqrt{-63}$, the difference is

(1) i (3) $-i\sqrt{7}$

(2) $i\sqrt{7}$ (4) $\sqrt{-35}$

32 The roots of $2x^2 - 3x + c = 0$ are imaginary if c equals

(1) 1 (3) -1

(2) 2 (4) 0

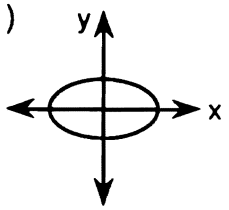
33 Every year a band is paid \$350 to play at the county fair. Let a represent the amount each member receives and let n represent the number of members in the band. The inverse variation relationship between a and n is best represented as

(1) $\frac{350}{a} = \frac{1}{n}$ (3) $a + n = 350$

(2) $\frac{n}{a} = 350$ (4) $an = 350$

34 Which diagram is *not* the graph of a function?

(1)



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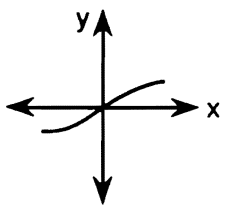

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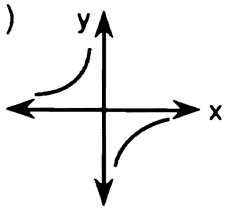

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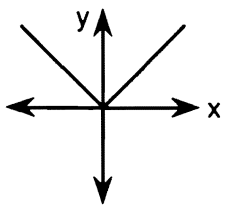


35 Which set of numbers forms a field with respect to the operations of addition and multiplication?

(1) integers (3) whole numbers

(2) rational numbers (4) natural numbers

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = 2 \cos x$ and $y = \sin 2x$ as x varies from $-\pi$ to π radians. [8]

b Use the graphs drawn in part *a* to determine all values of x in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $2 \cos x = \sin 2x$. [2]

37 *a* Express a in simplest radical form:

$$\frac{2}{a^2 - 7a + 10} = \frac{3}{a - 5} - \frac{a}{a - 2}$$ [6]

b Solve for x: $\log_{10}(x + 3) - \log_{10} x = 2$ [4]

38 Triangle RST has coordinates $R(2,-3)$, $S(1,1)$, and $T(-2,-1)$

a On graph paper, graph and label $\triangle RST$. [1]

b Graph and state the coordinates of $\triangle R'S'T'$, the image of $\triangle RST$ after $\left(D_{\frac{2}{3}} \circ D_3\right)$. [3]

c Graph and state the coordinates of $\triangle R''S''T''$, the image of $\triangle RST$ after $T_{5,-4}$. [3]

d Graph and state the coordinates of $\triangle R'''S'''T'''$, the image of $\triangle RST$ after $\left(r_{y\text{-axis}} \circ r_{x=4}\right)$. [3]

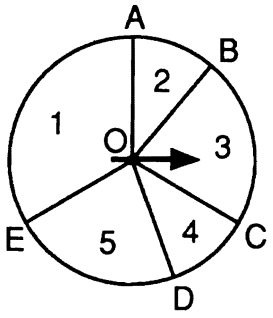
39 Two forces act on a body at an angle of 120°. The forces are 28 pounds and 35 pounds.

a Find the magnitude of the resultant force to the *nearest tenth* of a pound. [6]

b Find the angle formed by the greater of the two forces and the resultant force to the *nearest degree*. [4]

40 Find, to the *nearest degree*, all values of x in the interval $0° \leq x < 360°$ that satisfy the equation $3 \cos 2x + 5 \sin x - 2 = 0$. [10]

41 In the accompanying diagram of circle O, $m\overset{\frown}{AE} = m\overset{\frown}{AC} = m\overset{\frown}{EC}$ and $m\overset{\frown}{AB} = m\overset{\frown}{CD} = 40$.



a If the spinner is spun once, determine
(1) $P(1)$ [2]
(2) $P(\text{a number} \geq 2)$ [2]

b If the spinner is spun three times, determine the probability of obtaining
(1) *at most* one number ≥ 2 [3]
(2) *at least* one number ≥ 2 [3]

42 Given: $f(x) = x^2 - 4,\ x \geq 0$

a State the range of $f(x)$. [2]

b On graph paper, sketch $f(x)$ over its domain. Label the graph $f(x)$. [2]

c On the same set of axes, sketch $f^{-1}(x)$, the inverse of $f(x)$. Label the graph $f^{-1}(x)$. [2]

d Write an equation for $f^{-1}(x)$. [2]

e Find $(f \circ f^{-1})(5)$. [2]