HIGH SCHOOL MATHEMATICS: COURSE III-AUGUST 1981 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided in the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

- 1. Express 30° in radian measure.
- 2. Express cos (-310°) as a function of a positive acute angle.
- 3. Express in terms of i the sum $2\sqrt{-50} + \sqrt{-32}$.
- 4. If a function f is defined by $f(x) = 2x x^2$, find the value of f(4).
- 5. What are the coordinates of the point P(-2, 3) under the dilation D_2 ?
- 6. Two chords \overline{AB} and \overline{CD} intersect in circle O at point E. If CE = 6, ED = 5, and AE = 2, find EB.
- 7. What is the value of tan 23° 38' to four decimal places?
- 8. Solve for the positive value of x: $\frac{2x}{3\sqrt{2}} = \frac{3\sqrt{2}}{x}$
- 9. Find the middle term of the expansion of $(2x y)^4$.
- 10. If B(-2, 5) is reflected over the line y = x, what are the coordinates of the image of B?
- 11. If $\log_x 2 = \frac{1}{3}$, find x.
- 12. Express $\frac{1}{3+\sqrt{2}}$ as a fraction with a rational denominator.
- 13. In triangle ABC, $m \angle A = 30$, side b = 10, and side c = 20. Find the area of triangle ABC.
- 14. In triangle ABC, a = 8, b = 12, and $\sin A = \frac{1}{3}$. Find the value of $\sin B$.
- 15. In tossing a loaded die, the probability of getting a six is $\frac{1}{3}$. If the die is tossed 4 times, what is the probability of getting at least 3 sixes?
- 16. If T is the transformation $(x, y) \longrightarrow (-x, 2y)$, find the image of P(-3, 4) after the transformation T.
- 17. If $10^{2.5729} = 374$, find the value of $10^{0.5729}$.
- 18. In a circle, a central angle of 1 radian intercepts an arc of 2 centimeters. What is the length in centimeters of the radius of this circle?
- 19. Evaluate: $\sum_{k=4}^{6} (k^2 8)$
- 20. If $f(x) = 2x^{-\frac{1}{2}}$, find f(16).
- 21. If $f(x) = \cos x$, find the numerical value of $f\left(\frac{5\pi}{3}\right)$.

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- 22. Find the largest integral value of k in the equation $kx^2 5x + 3 = 0$ that will make its roots real.
- 23. If $\cos A = \frac{1}{2}$ and angle A is a positive acute angle, find the value of $\sin \frac{1}{2} A$.

Directions (24-35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

24. Which figure has both vertical and horizontal line symmetry?



- 25. If $\cos x \neq -1$, the fraction $\frac{1-\cos^2 x}{1+\cos x}$ is equivalent to
 - (1) $1 \cos x$ (2) $-\cos x$ (3) $1 + \cos x$ (4) $\cos x 1$
- 26. The value of $\sin \left(\operatorname{Arc} \cot \frac{4}{3} \right)$ is
 - (1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{3\sqrt{7}}{7}$ (4) $\frac{4}{3}$
- 27. A quadratic equation having the roots (2-3i) and (2+3i) is
 - (1) $x^2 + 4x + 13 = 0$ (2) $x^2 4x + 13 = 0$

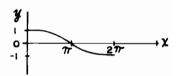
(3) $x^2 + 4x - 9 = 0$ (4) $x^2 - 4x + 9 = 0$

- 28. What is the range of the function $y = \sqrt{x-1}$ where $x \ge 1$?
 - (1) $y \ge 1$ (2) $y \ge 0$ (3) $y \le 0$ (4) all real numbers
- 29. In a standardized test, approximately what percent of the scores will fall between +1 standard deviation and -1 standard deviation from the mean?
 - (1) 5% (2) 34% (3) 68% (4) 95%
- 30. What is the solution set of the equation |5 2x| = 7?

 (1) $\{6, -1\}$ (2) $\{6\}$ (3) $\{-1\}$ (4) $\{$
- 31. If a figure has 180° rotational symmetry, it must have
 - (1) point symmetry
 - (2) horizontal line symmetry, only
 - (3) vertical line symmetry, only
 - (4) horizontal and vertical line symmetry
- 32. If $\cos x = \frac{3}{5}$ and angle x lies in the fourth quadrant, what is the value of $\tan x$?
 - (1) $\frac{5}{4}$ (2) $-\frac{5}{4}$ (3) $\frac{3}{4}$ (4) $-\frac{4}{3}$
- 33. If $\sin \alpha = \frac{3}{5}$, $\tan \beta = \frac{5}{12}$, and α and β are in the first quadrant, then the value of $\cos (\alpha + \beta)$ is
 - (1) $-\frac{16}{65}$ (2) $\frac{33}{65}$ (3) $\frac{56}{65}$ (4) $\frac{63}{65}$

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- 34. Which is the equation of the graph shown?
- (1) $y = \sin \frac{1}{2} x$ (2) $y = \cos \frac{1}{2} x$ (3) $y = \frac{1}{2} \sin x$
- (4) $y = \frac{1}{2} \cos x$



- 35. If $\sin A < 0$, in which quadrants may angle A lie?
 - (1) I, II (2) II, III (3) II, IV (4) III, IV

Answers to the following questions are to be placed on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 36. a. On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \frac{1}{2} \cos 2x$ as x varies from 0 to 2π radians. [8]
 - b. State the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $\sin x = \frac{1}{2} \cos 2x.$
- 37. The table below shows the weights of ten girls from a ninth-grade class. Determine the standard deviation of these weights to the nearest tenth.

Measure of Weight (x_i)	Frequency (f _i)
91	1
96	1
105	2
111	3
113	2
114	1

38. In the accompanying diagram, \overline{AOB} is a diameter of circle O, \overline{FBA} and \overline{FCE} are secants, chords \overline{AD} and \overline{CE} intersect at G, and

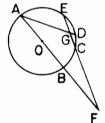
 $\widehat{\mathsf{mAE}}: \widehat{\mathsf{mED}}: \widehat{\mathsf{mDC}}: \widehat{\mathsf{mBC}} = 4:2:1:2.$

Find:

a. $m\widehat{AE}$ [2] d. m∠DAB

b. m \widehat{BC} [2] e. m∠AGC

c. m∠EFA [2]



- 39. a. Find, to the nearest tenth, the value(s) of tan θ that satisfy the equation tan² θ - $3 \tan \theta + 1 = 0$.
 - b. Using the answer(s) obtained in part a, find the quadrant(s) in which angle θ may lie. [2]

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- 40. The sides of triangle ABC are a = 10, b = 12, and c = 18. Find, to the nearest degree, the measure of the largest angle of triangle ABC. [10]
- 41. a. On graph paper, graph and label triangle ABC whose vertices have coordinates A(-2, 3), B(-6, 7), and C(-3, 8).
 - b. Graph and state the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$, after a reflection over the v-axis.
 - c. Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$, after a reflection over the line y = x. [3]
 - d. The composite $r_{v=x} \circ r_{v-axis}(\triangle ABC) = \triangle A''B''C''$ is a
 - (1) rotation (2) translation (3) dilation (4) glide reflection [1] e. Graph and state the coordinates of $\Delta A^{\prime\prime\prime}B^{\prime\prime\prime}C^{\prime\prime\prime}$, the image of ΔABC , the original triangle, after the transformation $(x, y) \longrightarrow (2x, -y)$. [3]
- 42. a. Using logarithms, solve the equation $4^x = 70$ for x to the nearest tenth. [5]
 - b. Solve the equation $x^2 = 6x 25$ and express its roots in the form a + bi. [5]