The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 23, 1999 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$\cot^2 A + 1 = \csc^2 A$

$\tan A = \dfrac{\sin A}{\cos A}$

$\cot A = \dfrac{\cos A}{\sin A}$

Functions of the Sum of Two Angles

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Functions of the Difference of Two Angles

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Law of Sines

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

Functions of the Double Angle

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Functions of the Half Angle

$\sin \frac{1}{2}A = \pm \sqrt{\dfrac{1 - \cos A}{2}}$

$\cos \frac{1}{2}A = \pm \sqrt{\dfrac{1 + \cos A}{2}}$

$\tan \frac{1}{2}A = \pm \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Area of Triangle

$K = \frac{1}{2}ab \sin C$

Standard Deviation

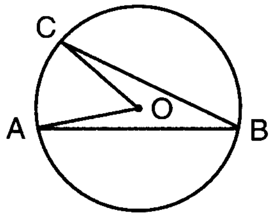
$\text{S.D.} = \sqrt{\dfrac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 Solve for x: $\sqrt{3x-8} + 4 = 11$

2 In the accompanying figure of circle O, $m\angle AOC = 52$. Find $m\angle ABC$.



3 Find the value of $\sum_{n=1}^{5} 2n$.

4 If k is a positive integer, what is the greatest value of k that will make $\sqrt{k-4}$ an imaginary number?

5 If $8^{x+1} = 4^{2x}$, what is the value of x?

6 Express 405° in radian measure.

7 In circle O, a central angle of 3 radians intercepts an arc of 27 meters. Find the number of meters in the length of the radius.

8 Solve for all values of x: $|2x + 5| = 4$

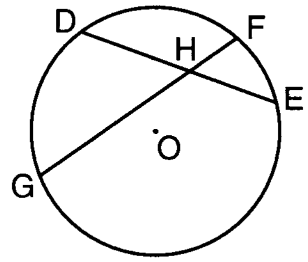
9 If $f(x) = \sin \frac{1}{2}x + 2 \cos x$, evaluate $f(\pi)$.

10 In $\triangle ABC$, $\cos C = -0.2$, $a = 8$, and $b = 10$. Find the length of side c.

11 Evaluate: $-3x^0 + (8)^{\frac{2}{3}} + \left(\frac{1}{2}\right)^{-2}$

12 If $\cos(2x - 25)° = \sin 55°$, find the value of x.

13 In the accompanying diagram, chords $\overline{DE}$ and $\overline{FG}$ intersect at H. If $DE = 18$, $HE = 8$, and $HF = 5$, find GH.



14 If $\sin x = \frac{2}{3}$, find the value of $\cos 2x$ in simplest fractional form.

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 If $f(x) = 3x^2$ and $g(x) = \sqrt{2x}$, what is the value of $(f \circ g)(8)$?

(1) $8\sqrt{6}$ (3) 48
(2) 16 (4) 144

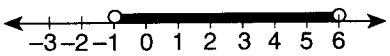
16 The value of $\cos\left(\text{Arc} \sin \frac{\sqrt{3}}{2}\right)$ is

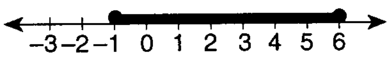
(1) 1 (3) $\frac{\sqrt{3}}{3}$
(2) $\frac{1}{2}$ (4) $\sqrt{3}$

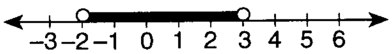
17 The expression $\log \frac{x^2y^3}{\sqrt{z}}$ is equivalent to

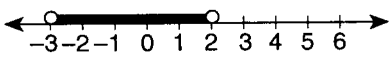
(1) $\frac{(2x)(3y)}{\frac{1}{2}z}$
(2) $2 \log x + 3 \log y + \frac{1}{2} \log z$
(3) $\log 2x + \log 3y - \log \frac{1}{2}z$
(4) $2 \log x + 3 \log y - \frac{1}{2} \log z$

18 Which graph represents the inequality $x^2 - 5x - 6 < 0$?

(1) 

(2) 

(3) 

(4) 

19 If $\log_2 (x^2 - 1) = \log_2 8$, then the solution set for x is

(1) $\{3,-3\}$ (3) $\{3\}$
(2) $\{-3\}$ (4) $\{\ \}$

20 In $\triangle ABC$, $\sin A = \frac{1}{2}$, $b = 20$, and $m\angle B = 45$. What is the length of side a?

(1) $\frac{10\sqrt{3}}{3}$ (3) $10\sqrt{2}$
(2) 10 (4) $20\sqrt{2}$

21 Expressed in simplest form, $\csc \theta \bullet \tan \theta \bullet \cos \theta$ is equivalent to

(1) 1 (3) $\cos \theta$
(2) $\sin \theta$ (4) $\tan \theta$

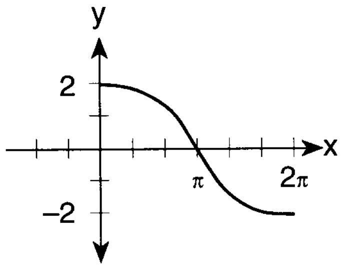
22 For which value of x is the function $f(x) = \frac{1}{1 - \tan x}$ undefined?

(1) 0 (3) $\frac{\pi}{3}$
(2) π (4) $\frac{\pi}{4}$

23 The expression $\dfrac{\frac{x}{z} - \frac{z}{x}}{\frac{1}{z} + \frac{1}{x}}$ is equivalent to

(1) $x - z$ (3) xz
(2) $x + z$ (4) $\frac{x-z}{xz}$

24 Which equation is represented in the graph below?



(1) $y = 2 \sin \frac{1}{2}x$ (3) $y = 2 \cos \frac{1}{2}x$
(2) $y = \frac{1}{2} \sin 2x$ (4) $y = \frac{1}{2} \cos 2x$

25 The expression $i^2(2 - i)$ is equivalent to

(1) $-2 - i$ (3) $2 - i$
(2) $-2 + i$ (4) $2 + i$

26 A spinner is divided into five equal sectors labeled 1 to 5. What is the probability of spinning exactly 3 even numbers in 4 spins?

(1) ${}_5C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)$ (3) ${}_5C_4\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)$
(2) ${}_4C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)$ (4) ${}_4C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)$

27 The roots of the equation $ax^2 + 4x = -2$ are real and equal if a has a value of

(1) 1 (3) 3
(2) 2 (4) 4

28 If the graph of the complex number $-2 + 5i$ is rotated counterclockwise 90° about the origin, the image will fall in Quadrant

(1) I (3) III
(2) II (4) IV

29 If $2\sqrt{-2}$ is subtracted from $3\sqrt{-18}$, the difference is

(1) $7i\sqrt{2}$ (3) $-7i\sqrt{2}$
(2) $11i\sqrt{2}$ (4) $-11i\sqrt{2}$

30 Which value of θ satisfies the equation $2 \sin^2 \theta - 5 \sin \theta - 3 = 0$?

(1) 300° (3) 150°
(2) 210° (4) 30°

31 For which equation does the sum of the roots equal 3 and the product of the roots equal 4.5?

(1) $x^2 + 3x - 9 = 0$ (3) $2x^2 + 6x + 9 = 0$
(2) $x^2 - 3x + 9 = 0$ (4) $2x^2 - 6x + 9 = 0$

32 If θ is an angle in standard position and its terminal side passes through point $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ on the unit circle, then a possible value of θ is

(1) 60° (3) 150°
(2) 120° (4) 330°

33 The expression cot (–200°) is equivalent to

(1) –tan 20° (3) –cot 20°
(2) tan 70° (4) cot 70°

34 A function is defined by the equation $y = 2x + 3$. Which equation defines the inverse of this function?

(1) $y = \frac{1}{2}x + \frac{1}{3}$ (3) $y = -2x - 3$
(2) $x = \frac{1}{2}y - \frac{3}{2}$ (4) $y = \frac{1}{2}x - \frac{3}{2}$

35 The graph of the equation $2x^2 - 5y^2 = 10$ forms

(1) a circle (3) a hyperbola
(2) an ellipse (4) a parabola

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = -3 \cos x$ and $y = \frac{1}{2} \sin 2x$ in the interval $-\pi \le x \le \pi$. [8]

b Based on the graphs drawn in part *a*, find all values in the interval $-\pi \le x \le \pi$ that satisfy the equation $-3 \cos x = \frac{1}{2} \sin 2x$. [2]

37 *a* Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of A in the interval $0° \le A < 360°$ that satisfy the equation $4 \sin^2 A + 1 = \sin^2 A + 2$. [6]

b Solve for x and express the roots of the equation $8x^2 - 28x + 29 = 0$ in simplest $a + bi$ form. [4]

38 *a* On graph paper, sketch the triangle formed by points $A(3,-3)$, $B(-1,-5)$, and $C(5,-4)$. [1]

b On the same set of axes, graph and state the coordinates of

(1) $\triangle A'B'C'$, the image of $\triangle ABC$ after the rotation $R_{90°}$ [3]

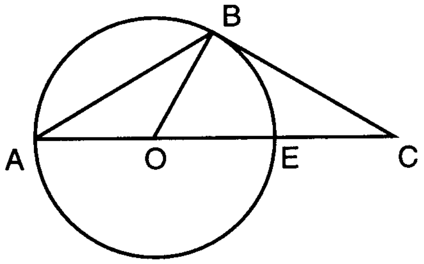
(2) $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after the translation $T_{-4,-1}$ [2]

(3) $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after the dilation D_3 [2]

c Is the composite transformation $\triangle ABC \rightarrow \triangle A'''B'''C'''$ an isometry? Explain your answer. [1,1]

GO RIGHT ON TO THE NEXT PAGE. ⇨

39 In the accompanying diagram of circle O, diameter $\overline{AE}$ is extended through E to C; tangent $\overline{CB}$, chord $\overline{AB}$, and radius $\overline{OB}$ are drawn; and $m\widehat{AB}:m\widehat{BE} = 2:1$.



a Find:

(1) $m\widehat{AB}$ [2]
(2) $m\angle BAC$ [2]
(3) $m\angle C$ [2]
(4) $m\angle ABC$ [2]

b Which term describes $\triangle OBC$? [1]

(1) acute
(2) right
(3) obtuse
(4) equiangular

c Explain how you arrived at your answer for part *b*. [1]

40 *a* In the equation $x^2 - 3x + c = 0$, one value of x is 2.5.

Find:

(1) c [2]
(2) the other value of x [3]

b Expand and express $(i - 3)^4$ in simplest $a + bi$ form, where i is the imaginary unit. [5]

41 *a* The accompanying table represents the PSAT scores of a group of ten students.

Score	*Frequency*
48	1
50	3
53	1
54	2
57	1
62	1
68	1

(1) Find the standard deviation to the *nearest tenth*. [4]
(2) How many scores fall within one standard deviation of the mean? [2]

b The probability of a biased coin coming up tails is $\frac{1}{4}$. When the coin is flipped four times, what is the probability of obtaining *at least* 2 tails? [4]

42 Michael and his friends are plotting a course for a race. They decided to make the course in the shape of a triangle, PQR. Beginning at point P, participants run 1.4 miles to Q, then from Q to R, and finally 2.6 miles from R back to P. Angle QPR measures 38°30'.

a Find, to the *nearest tenth of a mile*, the total number of miles for the entire race. [7]

b Find, to the *nearest tenth of a square mile*, the area of triangle PQR. [3]

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, June 23, 1999 — 1:15 to 4:15 p.m., only

Part I Score	....
Part II Score	....
Total Score	....
Rater's Initials:	....

ANSWER SHEET

Pupil .... Sex: ☐ Male ☐ Female Grade ....

Teacher .... School ....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 ....
2	12	22	32 ....
3	13	23	33 ....
4	14	24	34 ....
5	15	25	35 ....
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

____

Signature

Tear Here

Tear Here

Tear Here

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

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SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 19	(11) 5	(21) 1	(31) 4
(2) 26	(12) 30	(22) 4	(32) 2
(3) 30	(13) 16	(23) 1	(33) 3
(4) 3	(14) $\frac{1}{9}$	(24) 3	(34) 4
(5) 3	(15) 3	(25) 2	(35) 3
(6) $\frac{9\pi}{4}$	(16) 2	(26) 2	
(7) 9	(17) 4	(27) 2	
(8) $-\frac{1}{2}, -\frac{9}{2}$	(18) 1	(28) 3	
(9) –1	(19) 1	(29) 1	
(10) 14	(20) 3	(30) 2	

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *b* $-\frac{\pi}{2}, \frac{\pi}{2}$ [2]

(37) *a* 35.3°, 144.7°, 215.3°, 324.7°
or [6]
35°20', 144°40', 215°20', 324°40'

b $\frac{7}{4} \pm \frac{3}{4}i$ [4]

(38) *b* (1) $A'(3,3)$, $B'(5,-1)$, $C'(4,5)$ [3]
(2) $A''(-1,2)$, $B''(1,-2)$, $C''(0,4)$ [2]
(3) $A'''(-3,6)$, $B'''(3,-6)$, $C'''(0,12)$ [2]

c No, it does not preserve lengths. [1,1]

(39) *a* (1) 120 [2]
(2) 30 [2]
(3) 30 [2]
(4) 120 [2]

b 2 [1]

c A radius and a tangent are perpendicular at the point of contact.
or [1]
Show m$\angle OBC$ = 90.

(40) *a* (1) 1.25 [2]
(2) 0.5 [3]

b $28 - 96i$ [5]

(41) *a* (1) 5.9 [4]
(2) 7 [2]

b $\frac{67}{256}$ [4]

(42) *a* 5.7 [7]
b 1.1 [3]