The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, June 19, 1998 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

Standard Deviation

$$S.D. = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 If $f(x) = (2x)^2$, find $f(-4)$.

2 Solve for the positive value of x: $x^{\frac{2}{3}} = 9$

3 Solve for x: $\dfrac{x-3}{5} + \dfrac{4x}{3} = 4$

4 Solve for x: $\sqrt{2x-8} - 1 = 5$

5 Evaluate: $\displaystyle\sum_{n=1}^{5} n^2$

6 Find the image of $A(-3,2)$ under a dilation with the center at the origin and a scale factor of -2.

7 In $\triangle ABC$, $\sin A : \sin B : \sin C = 4:5:6$. Find the value of c when $a = 10$.

8 Find the value of $\tan\left(\text{Arc sin }\dfrac{5}{6}\right)$.

9 If a fair coin is flipped three times, what is the probability of obtaining exactly two heads?

10 In the accompanying diagram of circle O, secants $\overline{CBA}$ and $\overline{CED}$ intersect at C. If $AC = 12$, $BC = 3$, and $DC = 9$, find EC.
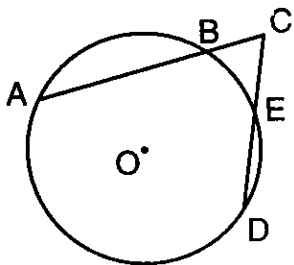
11 In a circle whose radius is 9 centimeters, what is the number of radians in a central angle if the length of the intercepted arc is 18 centimeters?

12 Find, in radical form, the area of $\triangle ABC$ if $a = 6$, $b = 6$, and $m\angle C = 45$.

13 Factor completely: $5x^2y^3 - 180y$

14 If P varies inversely as V and $P = 700$ when $V = 8$, find the value of V when $P = 350$.

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 In the accompanying diagram of circle O, $m\angle AOB = 80$.
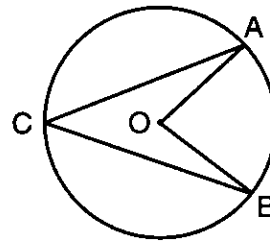
What is $m\angle ACB$?
(1) 80   (3) 20
(2) 160   (4) 40

16 The value of $\cos 16° \cos 164° - \sin 16° \sin 164°$ is
(1) -1 (3) 0
(2) $-\frac{1}{2}$ (4) $\dfrac{\sqrt{3}}{2}$

17 If the graphs of the equations $xy = 12$ and $y = 2$ are drawn on the same set of axes, what is the total number of common points?

(1) 1   (3) 3
(2) 2   (4) 0

18 The expression $\dfrac{\frac{a}{b} - 1}{\frac{a}{b} + 1}$ is equivalent to

(1) $\dfrac{a+b}{a-b}$ (3) $\dfrac{1}{a-b}$

(2) $\dfrac{a-b}{a+b}$ (4) $\dfrac{1}{a+b}$

19 The value of $\sin \dfrac{3\pi}{2} + \cos \dfrac{2\pi}{3}$ is

(1) $\frac{1}{2}$ (3) $-1\frac{1}{2}$

(2) $1\frac{1}{2}$ (4) $-\frac{1}{2}$

20 The sum of $3\sqrt{-8}$ and $4\sqrt{-50}$ is

(1) $12\sqrt{-58}$ (3) $7i\sqrt{58}$

(2) $26i\sqrt{2}$ (4) $7i\sqrt{2}$

21 What is the solution set of the inequality $|3x + 6| \le 30$?

(1) $-12 \le x \le 8$ (3) $x \le -12$ or $x \ge 8$
(2) $-8 \le x \le 12$ (4) $x \le -8$ or $x \ge 12$

22 The roots of the equation $x^2 + 6x + 11 = 0$ are

(1) real, rational, and unequal
(2) real, rational, and equal
(3) real, irrational, and unequal
(4) imaginary

23 If $\cos x = -\dfrac{\sqrt{2}}{2}$, in which quadrants could $\angle x$ terminate?

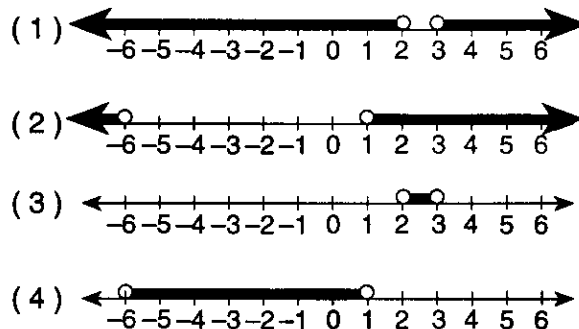(1) I and IV   (3) II and IV
(2) I and III   (4) II and III

24 Which expression is equivalent to $\dfrac{\sin 2x}{\cos x}$?

(1) $2 \sin x$ (3) $\cos 2x$
(2) $\tan x$ (4) $2 \cos x$

25 If $\sin (x - 3)° = \cos (2x + 6)°$, then the value of x is

(1) -9 (3) 29
(2) 26   (4) 64

26 Which graph represents the solution set of $x^2 + 5x - 6 > 0$?

(1)

(2)

(3)

(4)

27 What is the result of $T_{2,-1} \circ r_{y=-1} (2,0)$?

(1) $(2,0)$ (3) $(4,-3)$
(2) $(2,-1)$ (4) $(-4,3)$

28 In $\triangle ABC$, $a = 8$, $b = 2$, and $c = 7$. What is the value of $\cos C$?

(1) $-\dfrac{19}{32}$ (3) $\dfrac{109}{112}$

(2) $-\dfrac{11}{28}$ (4) $\dfrac{19}{32}$

29 What is the domain of $f(x) = \dfrac{1}{\sqrt{\left(4 - x^2\right)}}$?

(1) $x < 2$ (3) $-2 < x < 2$
(2) $|x| \le 2$ (4) all real numbers

30 In standard position, an angle of $\dfrac{7\pi}{3}$ radians has the same terminal side as an angle of

(1) 60°   (3) 240°
(2) 120°   (4) −420°

31 If the mean on a standardized test with a normal distribution is 54.3 and the standard deviation is 4.6, what is the best approximation of the percent of the scores that fall between 54.3 and 63.5?

(1) 34   (3) 68
(2) 47.5   (4) 95

32 In the accompanying diagram of a unit circle, the ordered pair (x,y) represents the point where the terminal side of θ intersects the unit circle.
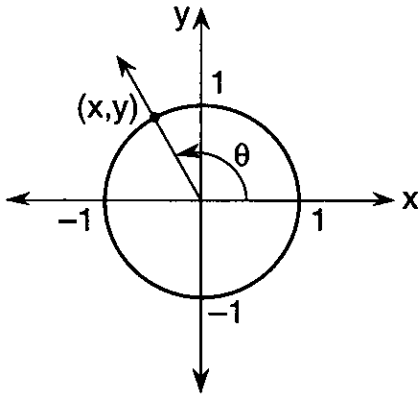
If $\theta = \dfrac{3\pi}{4}$, what is the value of x?

(1) 1

(3) $-\dfrac{\sqrt{2}}{2}$

(2) $-\dfrac{1}{2}$

(4) $\dfrac{\sqrt{3}}{2}$

33 What is the sum (S) and the product (P) of the roots of the equation $2x^2 - 4x + 1 = 0$?

(1) $S = \frac{1}{2}, P = 2$ (3) $S = -2, P = \frac{1}{2}$

(2) $S = 2, P = \frac{1}{2}$ (4) $S = -4, P = 1$

34 If $a = 5$, $c = 18$, and $m\angle A = 30$, what is the total number of distinct triangles that can be constructed?

(1) 1   (3) 3

(2) 2   (4) 0

35 What is the middle term in the expansion $(x - 3y)^4$?

(1) $54x^2y^2$ (3) $9x^2y^2$

(2) $54xy^2$ (4) $9xy^2$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = \sin \frac{1}{2}x$ and $y = -2\cos x$ in the interval $-\pi \le x \le \pi$. [8]

b Using the graphs drawn in part *a*, determine the number of solutions to the equation $\sin \frac{1}{2}x = -2\cos x$ in the interval $-\pi \le x \le \pi$. [2]

37 In the accompanying diagram of circle O, diameter $\overline{EOC}$ is extended through C to point P; diameter $\overline{AFOD}$, tangent $\overline{PD}$, and chords $\overline{AC}$, $\overline{CD}$, $\overline{BFE}$ are drawn; m$\angle COD = 60$; and m$\angle AFB = 100$.
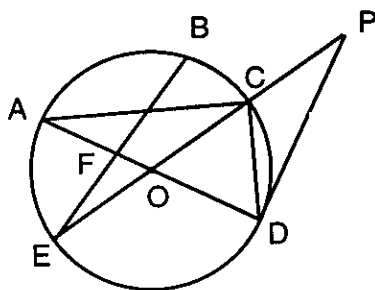
Find:

a m$\widehat{DE}$ [2]

b m$\angle P$ [2]

c m$\angle ACE$ [2]

d m$\widehat{AB}$ [2]

e m$\angle ACD$ [2]

38 *a* Given: $\log_b 3 = p$
            $\log_b 5 = q$

(1) Express $\log_b \frac{9}{5}$ in terms of p and q. [2]

(2) Express $\log_b \sqrt[3]{15}$ in terms of p and q. [2]

b Solve for x: $\log_4(x^2 + 3x) - \log_4(x + 5) = 1$ [6]

39 *a* Only red cards and black cards are in a box. The probability of drawing a black card is $\frac{3}{5}$. A card is randomly drawn and replaced in the box after each draw. Five such draws are made. Find the probability that

(1) *exactly* two black cards will be drawn [2]
(2) *at least* four black cards will be drawn [4]

b For these measurements, find the standard deviation, to the *nearest hundredth*:

85, 88, 79, 79, 80, 92, 94, 78, 80, 85 [4]

40 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $5 \sin^2 \theta - 7 \cos \theta + 1 = 0$. [10]

41 *a* Solve for x: $\dfrac{x}{x+5} + \dfrac{9}{x-5} = \dfrac{50}{x^2 - 25}$ [5]

b Solve for y and express the roots of the equation in simplest $a + bi$ form:

$$5y + \frac{5}{y} = 8 \quad [5]$$

42 *a* Given: $Z_1 = 4 - i$ and $Z_2 = -5 - 2i$.

(1) Using the complex plane, graph and label Z_1 and Z_2. [2]
(2) On the same plane, graph the sum of Z_1 and Z_2. [1]
(3) Express the sum of Z_1 and Z_2 as a complex number. [1]

b Forces of 40 pounds and 70 pounds act on a body at an angle measuring 60°. Find the magnitude of the resultant of these forces to the *nearest hundredth of a pound*. [6]

SEQUENTIAL MATH – COURSE III

Friday, June 19, 1998 — 1:15 to 4:15 p.m.. only

Part I Score	. . . .
Part II Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Pupil . . . .   Sex: ☐ Male ☐ Female   Grade . . . .

Teacher . . . .   School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

FOR TEACHERS ONLY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, June 19, 1998 — 1:15 to 4:15 p.m., only

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 64	(11) 2	(21) 1	(31) 2
(2) 27	(12) $9\sqrt{2}$	(22) 4	(32) 3
(3) 3	(13) $5y(xy + 6)(xy - 6)$	(23) 4	(33) 2
(4) 22	(14) 16	(24) 1	(34) 4
(5) 55	(15) 4	(25) 3	(35) 1
(6) (6,–4)	(16) 1	(26) 2	
(7) 15	(17) 1	(27) 3	
(8) $\dfrac{5}{\sqrt{11}}$	(18) 2	(28) 4	
(9) $\dfrac{3}{8}$	(19) 3	(29) 3	
(10) 4	(20) 2	(30) 1	

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *b* 2   [2]

(37) *a* 120   [2]
   b 30   [2]
   c 30   [2]
   d 80   [2]
   e 90   [2]

(38) *a* (1) $2p - q$ [2]

   (2) $\frac{1}{3}(p + q)$ [2]

   b −4,5   [6]

(39) *a* (1) $\frac{720}{3125}$ [2]

   (2) $\frac{1053}{3125}$ [4]

   b 5.48   [4]

(40) 53.1°, 306.9°
      or   [10]
   53°10', 306°50'

(41) *a* 1   [5]

   b $\frac{4}{5} \pm \frac{3}{5}i$ [5]

(42) *a* (3) $-1 - 3i$ [1]

   b 96.44   [6]