The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, June 19, 1997 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 Express 240° in radian measure.

2 In $\triangle ABC$, $a = 12$, $\sin A = 0.45$, and $\sin B = 0.15$. Find b.

3 Find the value of $\sum_{k=1}^{3}(3k - 5)$.

4 Solve for x: $4^{(3x+5)} = 16$

5 Express the sum of $\sqrt{-64}$ and $3\sqrt{-4}$ as a monomial in terms of i.

6 Solve for all values of x: $|2x + 5| = 7$

7 What will be the amplitude of the image of the curve $y = 2 \sin 3x$ after a dilation of scale factor 2?

8 What is the solution of the equation $\sqrt{5x - 9} - 3 = 1$?

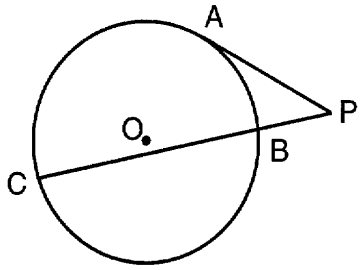
9 In the interval $90° \leq \theta \leq 180°$, find the value of θ that satisfies the equation $2 \sin \theta - 1 = 0$.

10 Express in simplest form: $\dfrac{1}{\frac{1}{a} + \frac{1}{b}}$

11 If $f(x) = x^0 + x^{\frac{2}{3}} + x^{-\frac{2}{3}}$, find $f(8)$.

12 When the sum of $4 + 5i$ and $-3 - 7i$ is represented graphically, in which quadrant does the sum lie?

13 In the accompanying diagram, $\overline{AP}$ is a tangent and $\overline{PBC}$ is a secant to circle O. If $PC = 12$ and $BC = 9$, find the length of $\overline{AP}$.



14 Circle O has a radius of 10. Find the length of an arc subtended by a central angle measuring 1.5 radians.

15 If $f(x) = 5x - 2$ and $g(x) = \sqrt[3]{x}$, evaluate $(f \circ g)(-8)$.

Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 For which value of x is the expression $\dfrac{1}{1 - \cos x}$ undefined?

(1) 90° (3) 270°
(2) 180° (4) 360°

17 The expression $\log \sqrt{\frac{x}{y}}$ is equivalent to

(1) $\frac{1}{2}(\log x - \log y)$ (3) $\frac{1}{2} \log x - \log y$
(2) $\log \frac{1}{2}x - \log \frac{1}{2}y$ (4) $\log \frac{1}{2}x - \log y$

18 If $f(x) = \cos 3x + \sin x$, then $f\left(\frac{\pi}{2}\right)$ equals

(1) 1 (3) −1
(2) 2 (4) 0

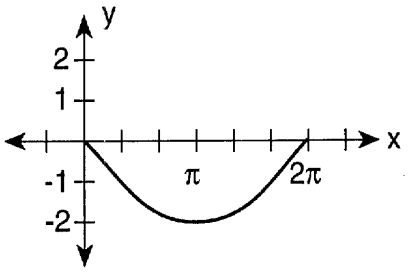
19 Expressed in $a + bi$ form, $(1 + 3i)^2$ is equivalent to

(1) $10 + 6i$
(2) $-8 + 6i$
(3) $10 - 6i$
(4) $-8 - 6i$

20 The expression $\frac{\tan \theta}{\sec \theta}$ is equivalent to

(1) $\cot \theta$
(2) $\csc \theta$
(3) $\cos \theta$
(4) $\sin \theta$

21 Which equation is represented by the graph below?



(1) $y = -2 \sin \frac{1}{2}x$
(2) $y = -\frac{1}{2} \sin 2x$
(3) $y = \frac{1}{2} \sin 2x$
(4) $y = 2 \sin \frac{1}{2}x$

22 Expressed in $a + bi$ form, $\frac{5}{3 + i}$ is equivalent to

(1) $\frac{15}{8} - \frac{5}{8}i$
(2) $\frac{5}{3} - 5i$
(3) $\frac{3}{2} - \frac{1}{2}i$
(4) $15 - 5i$

23 Gordon tosses a fair die six times. What is the probability that he will toss *exactly* two 5's?

(1) ${}_6C_5\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^4$
(2) ${}_6C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^4$
(3) ${}_6C_5\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$
(4) ${}_6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$

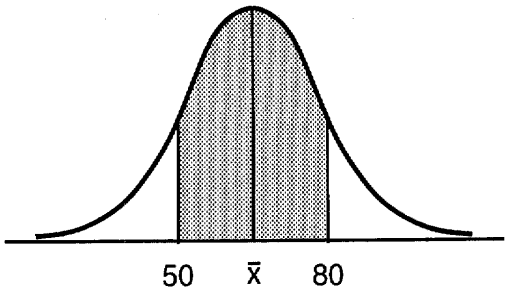
24 If $\sin \theta$ is negative and $\cot \theta$ is positive, in which quadrant does θ terminate?

(1) I
(2) II
(3) III
(4) IV

25 The domain of the equation $y = \frac{1}{(x - 1)^2}$ is all real numbers

(1) greater than 1
(2) except 1
(3) less than 1
(4) except 1 and –1

26 In the accompanying diagram, about 68% of the scores fall within the shaded area, which is symmetric about the mean, $\bar{x}$. The distribution is normal and the scores in the shaded area range from 50 to 80.



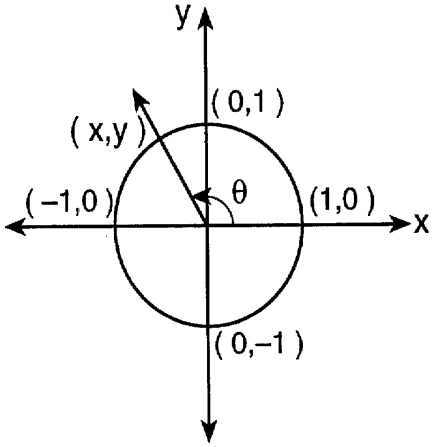
What is the standard deviation of the scores in this distribution?

(1) $7\frac{1}{2}$
(2) 15
(3) 30
(4) 65

27 The expression $2 \sin 30° \cos 30°$ has the same value as

(1) $\sin 15°$
(2) $\cos 60°$
(3) $\sin 60°$
(4) $\cos 15°$

28 In the accompanying diagram of a unit circle, the ordered pair (x,y) represents the point where the terminal side of θ intersects the unit circle.



If $m\angle\theta = 120$, what is the value of x in simplest form?

(1) $-\frac{\sqrt{3}}{2}$
(2) $\frac{\sqrt{3}}{2}$
(3) $-\frac{1}{2}$
(4) $\frac{1}{2}$

29 In $\triangle ABC$, side a is twice as long as side b and $m\angle C = 30$. In terms of b, the area of $\triangle ABC$ is

(1) $0.25\,b^2$
(2) $0.5\,b^2$
(3) $0.866\,b^2$
(4) b^2

30 Which quadratic equation has roots $3 + i$ and $3 - i$?

(1) $x^2 - 6x + 10 = 0$
(2) $x^2 + 6x - 10 = 0$
(3) $x^2 - 6x + 8 = 0$
(4) $x^2 + 6x - 8 = 0$

31 Which is the fourth term in the expansion of $(\cos x + 3)^5$?

(1) $90\cos^2 x$
(2) $270\cos^2 x$
(3) $90\cos^3 x$
(4) $270\cos^3 x$

32 The graph of the equation $y = \frac{6}{x}$ forms

(1) a hyperbola
(2) an ellipse
(3) a parabola
(4) a straight line

33 The roots of the equation $-3x^2 = 5x + 4$ are

(1) real, rational, and unequal
(2) real, irrational, and unequal
(3) real, rational, and equal
(4) imaginary

34 Which equation does *not* represent a function?

(1) $y = 4$
(2) $y = x^2 - 4$
(3) $y = x - 4$
(4) $x^2 + y^2 = 4$

35 If the point $(2,-5)$ is reflected in the line $y = x$, then the image is

(1) $(5,-2)$
(2) $(-2,5)$
(3) $(-5,2)$
(4) $(-5,-2)$

Answers to the following questions are to be written on paper provided by the school.

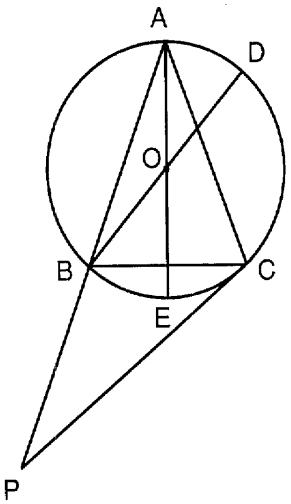
Part III

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = 2 \cos x$ and $y = \sin \frac{1}{2}x$ in the interval $-\pi \leq x \leq \pi$. [8]

b Using the graphs drawn in part *a*, determine the number of values in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $\sin \frac{1}{2}x = 2 \cos x$. [2]

37 In the accompanying diagram, isosceles triangle ABC is inscribed in circle O, and vertex angle BAC measures 40°. Tangent $\overline{PC}$, secant $\overline{PBA}$, and diameters $\overline{BD}$ and $\overline{AE}$ are drawn.



Find:

a $m\overset{\frown}{BC}$ [2]
b $m\angle ABD$ [2]
c $m\angle DOE$ [2]
d $m\angle P$ [2]
e $m\angle ACP$ [2]

38 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of x in the interval $0° \leq x < 360°$ that satisfy the equation $2 \sin 2x + \cos x = 0$. [10]

39 *a* Find the standard deviation, to the *nearest hundredth*, for the following measurements:

24,28,29,30,30,31,32,32,32,33,35,36 [4]

b A circle that is partitioned into five equal sectors has a spinner. The colors of the sectors are red, orange, yellow, blue, and green. If four spins are made, find the probability that the spinner will land in the green sector

(1) on *exactly* two spins [2]
(2) on *at least* three spins [4]

40 *a* Express in simplest form:

$$\frac{3y + 15}{25 - y^2} + \frac{2}{y - 5}$$ [5]

b Solve for x and express the roots in simplest $a + bi$ form:

$$2 + \frac{5}{x^2} = \frac{6}{x}$$ [5]

41 In $\triangle ABC$, $AB = 14$, $AC = 20$, and $m\angle CAB = 49$.

a Find the length of $\overline{BC}$ to the *nearest tenth*. [6]

b Using the results from part *a*, find $m\angle C$ to the *nearest degree*. [4]

42 Given: $f = \{(x,y) \mid y = \log_2 x\}$

a On graph paper, sketch and label the graph of the function f. [3]

b Write a mathematical explanation of how to form the inverse of function f. [2]

c On the same set of axes, sketch and label the graph of the function f^{-1}, the inverse of f. [3]

d Write an equation for f^{-1}. [2]

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, June 19, 1997 — 1:15 to 4:15 p.m., only

Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Pupil ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 .....
2	12	22	32 .....
3	13	23	33 .....
4	14	24	34 .....
5	15	25	35 .....
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

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SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $\frac{4\pi}{3}$

(2) 4

(3) 3

(4) –1

(5) $14i$

(6) –6, 1

(7) 4

(8) 5

(9) 150°

(10) $\frac{ab}{b+a}$

(11) $5\frac{1}{4}$

(12) IV

(13) 6

(14) 15

(15) –12

(16) 4

(17) 1

(18) 1

(19) 2

(20) 4

(21) 1

(22) 3

(23) 4

(24) 3

(25) 2

(26) 2

(27) 3

(28) 3

(29) 2

(30) 1

(31) 2

(32) 1

(33) 4

(34) 4

(35) 3

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *b* 2 [2]

(37) *a* 80 [2]

b 20 [2]

c 140 [2]

d 30 [2]

e 110 [2]

(38) 90°, 194.5°, 270°, 345.5°
or
90°, 194°30′, 270°, 345°30′ [10]

(39) *a* 3.06 [4]

b (1) $\frac{96}{625}$ [2]

(2) $\frac{17}{625}$ [4]

(40) *a* $\frac{1}{5-y}$ [5]

b $\frac{3}{2} \pm \frac{1}{2}i$ [5]

(41) *a* 15.1 [6]

b 44 [4]

(42) *b* Reflect the function in $y = x$
or
Interchange the x's and y's [8]
or
An equivalent statement

d $y = 2^x$
or [2]
$x = \log_2 y$