The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 19, 1996 – 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9, you will find a formula sheet which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

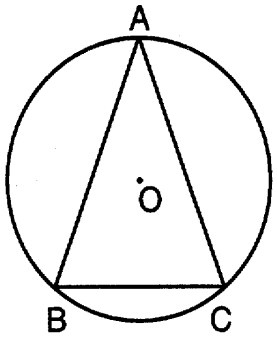
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 If $f(x) = \sqrt{25 - x^2}$, find the value of f(3).

2 An angle that measures $\frac{5\pi}{6}$ radians is drawn in standard position. In which quadrant does the terminal side of the angle lie?

3 In the accompanying diagram, isosceles triangle ABC is inscribed in circle O and $m\angle BAC = 40$. Find $m\overset{\frown}{AC}$.



4 Solve for x: $\log_x 125 = 3$

5 Point (–3,4) is rotated 180° about the origin in a counterclockwise direction. What are the coordinates of its image?

6 For which positive value of x is the function $f(x) = \frac{5x}{x^2 - 4x - 45}$ undefined?

7 Solve for x: $8^x = 2^{(x+6)}$

8 If $h(x) = 2x - 1$ and $g(x) = 3x + 1$, what is $(h \circ g)(2)$?

9 Subtract $(3 - 2i)$ from $(-2 + 3i)$, and express in $a + bi$ form.

10 In $\triangle ABC$, $a = 8$, $b = 7$, and $m\angle C = 30$. What is the area of $\triangle ABC$?

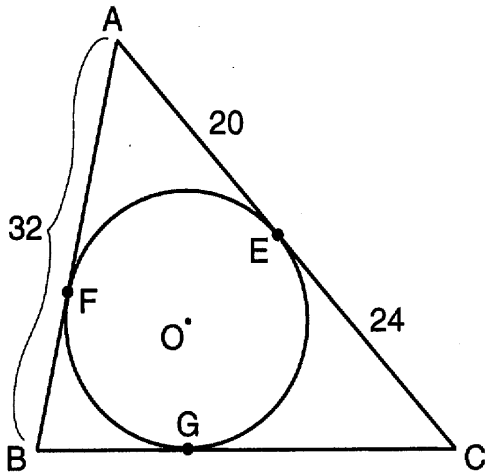
11 Evaluate: $\sum_{r=1}^{3} r^{(r-1)}$

12 Chords $\overline{XY}$ and $\overline{ZW}$ intersect in a circle at P. If $XP = 7$, $PY = 12$, and $WP = 14$, find PZ.

13 Find the number of degrees in the measure of the *smallest* positive angle that satisfies the equation $2 \cos x + 1 = 0$.

14 Find the complete solution set of $|2x - 4| = 8$.

15 In the accompanying diagram, $\overline{AFB}$, $\overline{AEC}$, and $\overline{BGC}$ are tangent to circle O at F, E, and G, respectively. If $AB = 32$, $AE = 20$, and $EC = 24$, find BC.



Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 The expression $\frac{\sqrt{-36}}{-\sqrt{36}}$ is equivalent to

(1) $6i$
(2) i
(3) $-i$
(4) $-6i$

17 If $\sin\theta < 0$ and $\tan\theta = -\frac{4}{5}$, in which quadrant does θ terminate?

(1) I
(2) II
(3) III
(4) IV

18 The expression $\frac{\frac{a-1}{a}}{\frac{a^2-1}{a^2}}$ is equivalent to

(1) $\frac{a}{a+1}$
(2) $\frac{a+1}{a}$
(3) $\frac{a}{a-1}$
(4) $\frac{a-1}{a}$

19 The roots of the equation $x^2 + 7x - 8 = 0$ are

(1) real, rational, and equal
(2) real, rational, and unequal
(3) real, irrational, and unequal
(4) imaginary

20 The product of $(-2 + 6i)$ and $(3 + 4i)$ is

(1) $-6 + 24i$
(2) $-6 - 24i$
(3) $18 + 10i$
(4) $-30 + 10i$

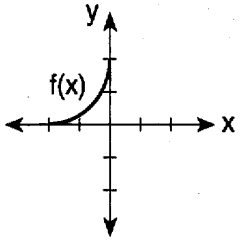
21 If $\sin 2A = \cos 3A$, then $m\angle A$ is

(1) $1\frac{1}{2}$
(2) 5
(3) 18
(4) 36

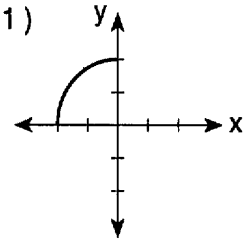
22 The graph of the equation $x = \frac{2}{y}$ is best described as

(1) a circle
(2) an ellipse
(3) a hyperbola
(4) a parabola

23 The accompanying diagram represents the graph of $f(x)$.



Which graph below represents $f^{-1}(x)$?

(1)


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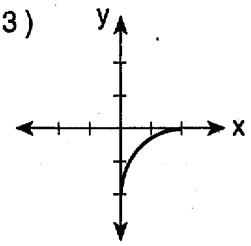

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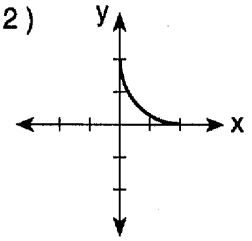

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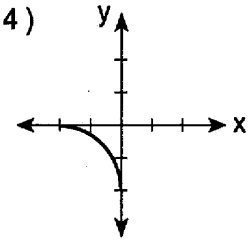


24 What is the domain of $f(x) = \sqrt{x-4}$ over the set of real numbers?

(1) $\{x|x \leq 4\}$
(2) $\{x|x \geq 4\}$
(3) $\{x|x > 4\}$
(4) $\{x|x = 4\}$

25 What is the solution set of the equation $\sqrt{5-x} + 3 = x$?

(1) {1}
(2) {4,1}
(3) { }
(4) {4}

26 If $\log 28 = \log 4 + \log x$, what is the value of x?

(1) 7
(2) 14
(3) 24
(4) 32

27 If $a = 4$, $b = 6$, and $\sin A = \frac{3}{5}$ in $\triangle ABC$, then $\sin B$ equals

(1) $\frac{3}{20}$ (3) $\frac{8}{10}$

(2) $\frac{6}{10}$ (4) $\frac{9}{10}$

28 What is the image of (5,–2) under the transformation $r_{y=x}$?

(1) (–5,2) (3) (2,5)

(2) (5,2) (4) (–2,5)

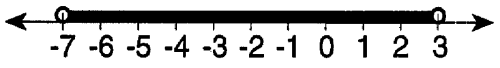
29 Each day the probability of rain on a tropical island is $\frac{7}{8}$. Which expression represents the probability that it will rain on the island exactly n days in the next 3 days?

(1) ${}_3C_n \left(\frac{7}{8}\right)^n \left(\frac{1}{8}\right)^{3-n}$ (3) ${}_nC_3 \left(\frac{7}{8}\right)^3 \left(\frac{1}{8}\right)^n$

(2) ${}_3C_3 \left(\frac{7}{8}\right)^3 \left(\frac{1}{8}\right)^n$ (4) ${}_8C_7 (3)^n (3)^{8-n}$

30 Which graph represents the solution of the inequality $x^2 + 4x - 21 < 0$?

(1)



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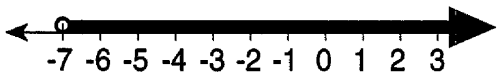

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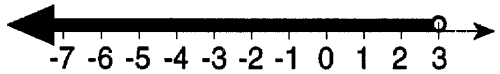

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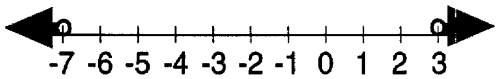


31 On a standardized test, the mean is 68 and the standard deviation is 4.5. What is the best approximation of the percentage of scores that will fall in the range 59–77?

(1) 34% (3) 95%

(2) 68% (4) 99%

32 In $\triangle ABC$, $a = 6$, $b = 4$, and $c = 9$. The value of $\cos C$ is

(1) $\frac{61}{72}$ (3) $\frac{2}{3}$

(2) $-\frac{29}{48}$ (4) $\frac{4}{9}$

33 If $m\angle A = 125$, $AB = 10$, and $BC = 12$, what is the number of distinct triangles that can be constructed?

(1) 1 (3) 3

(2) 2 (4) 0

34 The graph of which function has an amplitude of 2 and a period of 4π?

(1) $y = 2 \sin \frac{1}{2}x$ (3) $y = 4 \sin \frac{1}{2}x$

(2) $y = 2 \sin 4x$ (4) $y = 4 \sin 2x$

35 What is the sum of the roots of the equation $2x^2 + 6x - 7 = 0$?

(1) $-\frac{7}{2}$ (3) 3

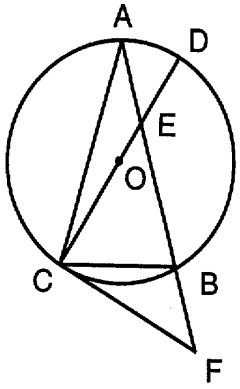
(2) –3 (4) $\frac{7}{2}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 In the accompanying diagram of circle O with inscribed isosceles triangle ABC, $\overline{AB} \cong \overline{AC}$, $m\widehat{CB} = 60$, $\overline{FC}$ is a tangent, and secant $\overline{FBA}$ intersects diameter $\overline{CD}$ at E.



Find:

a $m\angle ABC$ [2]

b $m\widehat{AD}$ [2]

c $m\angle DEB$ [2]

d $m\angle AFC$ [2]

e $m\angle BCF$ [2]

37 *a* On graph paper, sketch the graph of the equation $y = 2 \cos x$ in the interval $-\pi \leq x \leq \pi$. [4]

b On the same set of axes, reflect the graph drawn in part *a* in the x-axis and label it *b*. [2]

c Write an equation of the graph drawn in part *b*. [2]

d Using the equation from part *c*, find the value of y when $x = \frac{\pi}{6}$. [2]

38 Find, to the *nearest degree*, all values of x in the interval $0° \leq x < 360°$ that satisfy the equation $3 \cos 2x + \cos x + 2 = 0$. [10]

39 In a contest, the probability of the Alphas beating the Betas is $\frac{3}{5}$. The teams compete four times a season and each contest has a winner. Find the probability that

a the Betas win all four contests [2]

b each team wins two contests during the season [2]

c the Alphas win *at least* two contests during the season [3]

d the Betas win *at most* one contest during the season [3]

40 Answer both *a* and *b*.

a For all values of x for which the expressions are defined, prove that the following is an identity:

$$\tan x + \cot x = 2 \csc 2x$$ [6]

b Given: $\log 2 = x$ and $\log 3 = y$.

(1) Express $\log \frac{\sqrt{2}}{9}$ in terms of x and y. [2]

(2) Express $\log \sqrt[3]{6}$ in terms of x and y. [2]

☞ GO RIGHT ON TO THE NEXT PAGE.

41 Answer both *a* and *b*.

a Expand and express in simplest form:

$$\left(x - \frac{1}{x}\right)^4 \qquad [7]$$

b Solve for x to the *nearest tenth*:

$$5^{3x} = 1{,}000 \qquad [3]$$

42 The lengths of the sides of $\triangle ABC$ are 9.5, 12.8, and 13.7.

a Find, to the *nearest hundredth of a degree* or the *nearest ten minutes*, the measure of the *smallest* angle in the triangle. [6]

b Find, to the *nearest tenth*, the area of $\triangle ABC$. [4]

Tear Here

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, June 19, 1996 – 1:15 to 4:15 p.m., only

Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Pupil ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 .....
2	12	22	32 .....
3	13	23	33 .....
4	14	24	34 .....
5	15	25	35 .....
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 19, 1996 – 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 4

(2) II

(3) 140

(4) 5

(5) (3,–4)

(6) 9

(7) 3

(8) 13

(9) $-5 + 5i$

(10) 14

(11) 12

(12) 6

(13) 120

(14) –2,6

(15) 36

(16) 3

(17) 4

(18) 1

(19) 2

(20) 4

(21) 3

(22) 3

(23) 3

(24) 2

(25) 4

(26) 1

(27) 4

(28) 4

(29) 1

(30) 1

(31) 3

(32) 2

(33) 1

(34) 1

(35) 2

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* 75 [2]

b 30 [2]

c 135 [2]

d 45 [2]

e 30 [2]

(37) *c* $y = -2\cos x$ [2]

d $-\sqrt{3}$ [2]

(38) 71, 120, 240, 289 [10]

(39) *a* $\frac{16}{625}$ [2]

b $\frac{216}{625}$ [2]

c $\frac{513}{625}$ [3]

d $\frac{297}{625}$ [3]

(40) *b* (1) $\frac{1}{2}x - 2y$ [2]

(2) $\frac{1}{3}(x + y)$ [2]

(41) *a* $x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$ [7]

b 1.4 [3]

(42) *a* 41.84° *or* 41°50′ [6]

b 58.5 [4]