The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, June 19, 1992 — 1:15 to 4:15 p.m., only

Notice . . .

   Calculators must be available to all students taking this examination.

   The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

   The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet. Open the booklet and carefully remove the reference tables.

   When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may left in terms of π or in radical form. [60]

1 In $\triangle ABC$, $\sin C = \frac{1}{4}$, $c = 6$, and $a = 12$. Find $\sin A$.

2 Express $8\sqrt{-121} - 4\sqrt{-81}$ as a monomial in terms of i.

3 Evaluate: $\frac{1}{2}\sum_{x=1}^{4} (x - 1)^2$

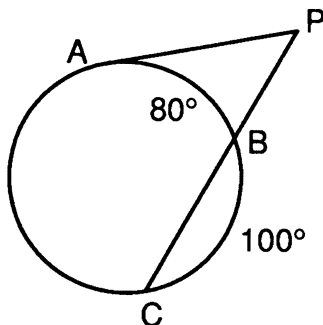4 In $\triangle ABC$, $a = 16$, $b = 30$, and $m\angle C = 150$. Find the area of $\triangle ABC$.

5 Given the function: $f(x) = (x + 1)^2$. What is the value of $f(-2)$?

6 For which value of x is the function $g(x) = \dfrac{2x^2 + 3x + 1}{x - 1}$ undefined?

7 Solve for x: $\sqrt{x + 10} + 2 = 5$

8 Solve for x: $2^{3x+3} = 8^{2x}$

9 In the diagram below, $\overline{PA}$ is tangent to the circle at A and $\overline{PBC}$ is a secant. If $m\widehat{AB} = 80$ and $m\widehat{BC} = 100$, what is $m\angle APB$?
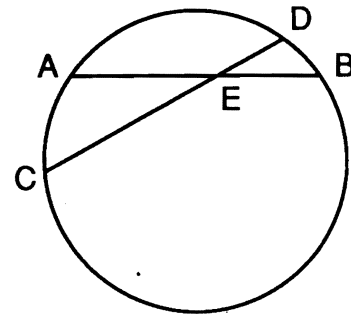

10 Solve for all values of x: $|6 - x| = 4$

11 In a circle, an inscribed angle intercepts an arc of $140°$. Find the number of degrees in the measure of the angle.

12 Find the value of $x^{-\frac{3}{2}}$ if $x = 16$.

13 Solve for x: $\dfrac{2}{x} + \dfrac{10}{3x} = 1$

14 In the accompanying diagram, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AB = 15$, $BE = 6$, and $CE = 12$, find DE.

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 The number of degrees equal to $\frac{4}{9}\pi$ radians is
(1) 60   (3) 130
(2) 80   (4) 270

16 Given: t varies inversely as p. If p is divided by 2, then t is
(1) increased by 2   (3) divided by 2
(2) decreased by 2   (4) multiplied by 2

17 If $\sin A > 0$ and $\cos A < 0$, in which quadrant does $\angle A$ terminate?
(1) I   (3) III
(2) II   (4) IV

18 What is the range of the function $y = 3 \sin x$?

(1) $y \geq 0$ (3) $y \leq 3$
(2) $-1 \leq y \leq 1$ (4) $-3 \leq y \leq 3$

19 The expression $\cos 2A - \cos^2 A$ is equivalent to

(1) $\cos^2 A + 1$ (3) $-\sin^2 A$
(2) $\sin^2 A - 1$ (4) $\cos^2 A$

20 The multiplicative inverse of $4 + i$ is

(1) $\dfrac{4 - i}{17}$ (3) $\dfrac{4 - i}{15}$

(2) $\dfrac{4 + i}{17}$ (4) $\dfrac{4 + i}{14}$

21 If $\cos x = 0.8$, what is a value of $\tan \frac{1}{2}x$?

(1) $\frac{1}{3}$ (3) 3

(2) $\frac{1}{9}$ (4) 9

22 What is the image of $A(3,4)$ under $R_{90°}$?

(1) $(3,-4)$ (3) $(-3,-4)$
(2) $(-4,3)$ (4) $(-4,-3)$

23 The value of $\sin 60° \cos 45° - \sin 45° \cos 60°$ is

(1) 1 (3) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

(2) 0 (4) $\frac{1}{2}$

24 The complex fraction $\dfrac{1 + \dfrac{1}{x}}{x - \dfrac{1}{x}}$, $x \neq -1, 0, 1$,

is equivalent to
(1) 1 (3) $\dfrac{1}{x - 1}$

(2) -1 (4) $\dfrac{1}{1 - x}$

25 A coin is biased so that the probability of tails is $\frac{1}{3}$. What is the probability of obtaining exactly two tails on five tosses of the coin?

(1) $_5C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$ (3) $_5C_2\left(\frac{1}{3}\right)^2$

(2) $_5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)^3$ (4) $2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$

26 What is the period of $y = 2 \sin 3x$?

(1) $\frac{2}{3}\pi$ (3) π

(2) 2 (4) 6π

27 The value of Arc sin (-1) is

(1) π (3) $\dfrac{\pi}{2}$

(2) $-\dfrac{\pi}{2}$ (4) $-\dfrac{\pi}{4}$

28 What is the value of $\cos (-240°)$?

(1) $\dfrac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$

(2) $-\dfrac{\sqrt{3}}{2}$ (4) $-\frac{1}{2}$

29 In a circle whose radius is 8, the length of an arc of the circle is 2π. What is the number of radians in the central angle subtended by the arc?

(1) 16π (3) $\dfrac{\pi}{4}$

(2) $\dfrac{\pi}{2}$ (4) 4π

30 If the distance from the origin to point A is $\sqrt{10}$ and $A' = D_3 (A)$, the distance from the origin to point A' is

(1) $3\sqrt{10}$ (3) $3 + \sqrt{10}$
(2) $2\sqrt{10}$ (4) $\dfrac{\sqrt{10}}{3}$

31 The equation $y = a^x$ expressed in logarithmic form is
(1) $y = \log_a x$ (3) $x = \log_a y$
(2) $a = \log_x y$ (4) $x = \log_y a$

32 On a standardized test with a standard deviation of 2, a score of 26 will occur fewer than 5 times out of 100. Which score could be the mean for this test?
(1) 20 (3) 24
(2) 23 (4) 25

33 The roots of the equation $3n^2 + 2n - 3 = 0$ are best described as
(1) real, rational, and unequal
(2) real, irrational, and unequal
(3) real, rational, and equal
(4) imaginary

34 What is the fourth term in the expansion of $\left(\dfrac{x}{2} + 2\right)^6$?

(1) $60x^2$

(2) $40x^3$

(3) $20x^3$

(4) $\dfrac{15x^4}{4}$

35 The graph of the equation $3y^2 = 6 - x^2$ is
(1) a hyperbola   (3) a parabola
(2) a circle   (4) an ellipse

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.   [40]

36 In the accompanying diagram of the circle, $\overline{PBA}$ and $\overline{PCD}$ are secants. chords $\overline{AC}$ and $\overline{BD}$ intersect at E, $\overline{BA} \cong \overline{CD}$. chord $\overline{BC}$ is drawn, $m\angle ABD = 60$. and $m\overset{\frown}{BC} = 40$.
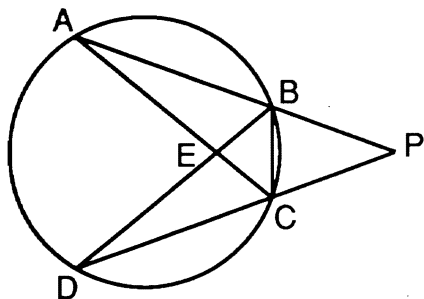
Find:

a	m∠ACD	[2]
b	m∠P	[2]
c	m∠DBC	[2]
d	m∠AED	[2]
e	m∠PCB	[2]

37 During a 10-week softball season, Jo got the following hits per week:
3. 5. 6. 1. 5. 7. 6. 5, 6, 3

a Find the standard deviation of the number of hits.   [6]

b The probability of Jo getting a hit is $\frac{1}{4}$. If she comes to bat three times, what is the probability that she will get at least two hits?   [4]

38 a Sketch and label the graph of the equation $y = 2^x$ in the interval $-3 \le x \le 3$.   [4]

b On the same set of axes, sketch and label the graph of the equation $xy = 8$ in the interval $-8 \le x \le 8$.   [4]

c Using the graphs drawn in parts a and b, solve for x: $2^x = \frac{8}{x}$.   [2]

39 a Sketch the graph of the equation $y = 2 \sin x$ in the interval $-\pi \le x \le \pi$.   [4]

b On the same set of axes, reflect the graph drawn in part a in the y-axis and label the graph b.   [2]

c Write an equation of the graph drawn in part b.   [2]

d Using the equation from part c, find the value of y when $x = \frac{\pi}{6}$.   [2]

40 One angle of a rhombus measures $100°$, and the longer diagonal measures 5.8 meters.

To the *nearest tenth* of a meter, find the length of

a a side of the rhombus   [6]

b the shorter diagonal   [4]

41 a Find, to the *nearest degree*, all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $\csc^2 \theta - 7 \csc \theta - 8 = 0$.   [6]
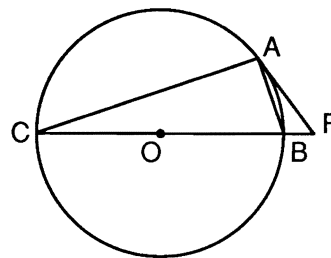
b Using logarithms, find x to the *nearest hundredth*:
$$11.6^x = 7.7 \quad [4]$$

42 a Prove that the following is an identity:
$$\frac{\cos 2\theta}{\sin \theta} + \sin \theta = \frac{1}{\sin \theta} - \sin \theta \quad [5]$$

b Diameter $\overline{CB}$ of circle O is extended to point P, $\overline{PA}$ is tangent to the circle at A, and chords $\overline{AB}$ and $\overline{AC}$ are drawn.

Prove that $\angle PAB \cong \angle ACB$.   [5]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, June 19, 1992 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $\frac{1}{2}$	(11) 70	(21) 1	(31) 3
(2) $52i$	(12) $\frac{1}{64}$	(22) 2	(32) 1
(3) 7	(13) $5\frac{1}{3}$	(23) 3	(33) 2
(4) 120	(14) $4\frac{1}{2}$	(24) 3	(34) 3
(5) 1	(15) 2	(25) 1	(35) 4
(6) 1	(16) 4	(26) 1	
(7) –1	(17) 2	(27) 2	
(8) 1	(18) 4	(28) 4	
(9) 50	(19) 3	(29) 3	
(10) 2, 10	(20) 1	(30) 1	

Part II

Please refer to the Department publication *Guide for Rating Regents Examinations in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Note: In October 1991, a supplement to the *Guide for Rating Regents Examinations in Mathematics* was sent to all schools. This supplement includes references to problem solving and calculator use. Teachers should become familiar with these modifications before rating student papers.

(36) *a* 60 [2]
    b 40 [2]
    c 50 [2]
    d 80 [2]
    e 70 [2]

(37) *a* 2 [6]
    b $\frac{10}{64}$ [4]

(38) *c* 2 [2]

(39) *c* $y = -2 \sin x$ [2]
    d -1 [2]

(40) *a* 3.8 [6]
    b 4.9 [4]

(41) *a* 7, 173, 270 [6]
    b 0.83 [4]