

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 19, 1991 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

If your school allows the use of calculators for this examination, they may be used for checking purposes **only**. In Part II, all work, including calculations, must be shown on your answer paper.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The “Reference Tables for Mathematics” and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet. Open the booklet and carefully remove the reference tables.

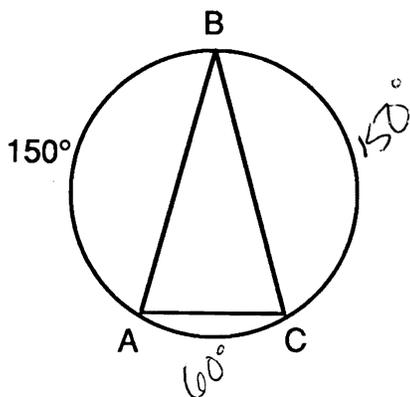
When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

- 1 In the accompanying diagram, isosceles triangle ABC is inscribed in the circle. If $\overline{AB} \cong \overline{CB}$ and $m\widehat{AB} = 150$, find $m\angle B$.



- 2 In a circle, a central angle of 3 radians intercepts an arc of 15 centimeters. Find the length of the radius in centimeters.

- 3 In $\triangle ABC$, $a = 6$, $b = 8$, and $\sin A = \frac{1}{4}$. What is the value of $\sin B$?

4 Solve for x : $\frac{3}{2x - 1} = \frac{1}{3x - 5}$

- 5 Express 160° in radian measure.

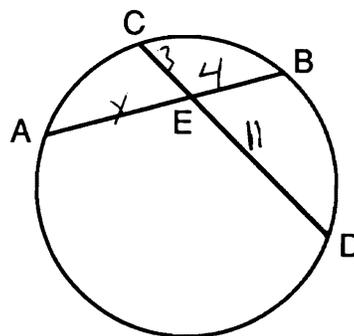
- 6 If $\sin \theta = -\frac{4}{5}$ and $\tan \theta$ is negative, what is the value of $\cos \theta$?

- 7 Express $5\sqrt{-18} + 6\sqrt{-98}$ as a monomial in terms of i .

- 8 Express $\frac{2}{3 - \sqrt{2}}$ as an equivalent fraction with a rational denominator.

- 9 Find the value of $\sum_{n=1}^3 (n + 1)(n - 1)$.

- 10 In the accompanying figure, chords \overline{AB} and \overline{CD} intersect at E . If $CD = 11$, $EB = 4$, and $CE = 3$, find AE .



- 11 Solve for x : $4^{x-1} = 2^x$

- 12 For which value of x is $f(x) = \frac{x^2 - 2x + 1}{3x - 1}$ undefined?

- 13 If x varies inversely as y and $x = 4$ when $y = 8$, find x when $y = 1$.

14 Evaluate: $4^0 - 8^{\frac{2}{3}} + 9^{\frac{1}{2}}$

- 15 If $\cos \theta = -\frac{3}{5}$, find $\cos 2\theta$ and express in simplest form.

- 16 In $\triangle ABC$, $AB = 8$, $AC = 6$, and $m\angle CAB = 150$. Find the area of the triangle.

- 17 If $f(x) = 2 \cos^2 x$, find $f\left(\frac{\pi}{6}\right)$.

- 18 Find the measure of the *smallest* positive angle that satisfies the equation $\tan^2 A - 3 = 0$.

Directions (19–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

19 For all values of x for which the expression is defined, $\sec x \cdot \csc x \cdot \cos x$ is equivalent to

- (1) $\tan x$ (3) $\frac{1}{\sin x}$
 (2) $\sin x$ (4) $\frac{1}{\cos x}$

20 If $x = u^2v$, which expression is equivalent to $\log x$?

- (1) $2 \log u + \log v$ (3) $\frac{2 \log u}{\log v}$ *$\log u^2v$*
 (2) $\log 2u + \log v$ (4) $2 \log u \log v$

21 In $\triangle ABC$, $a = 4$, $b = 3$, and $c = 3$. What is the value of $\cos A$?

- (1) $\frac{1}{18}$ (3) $\frac{1}{9}$
 (2) $-\frac{1}{18}$ (4) $-\frac{1}{9}$

22 If $4\sqrt{5} = \sqrt{n}$, the value of n is

- (1) 10 (3) 80
 (2) 20 (4) 100

23 The value of $\cos 64^\circ \cos 26^\circ - \sin 64^\circ \sin 26^\circ$ is

- (1) 1 (3) $\frac{\sqrt{3}}{2}$
 (2) $\frac{1}{2}$ (4) 0

24 A survey of coffee-drinking habits of the population in a small town revealed the mean number of cups drunk per week is 20 and the standard deviation is 3.5. If a normal distribution is assumed, how many cups per week will approximately 68% of the population of this small town drink?

- (1) 15 to 20 (3) 13 to 27
 (2) 16.5 to 23.5 (4) 0 to 30

25 Which is the equivalent exponential form of $\log_b N = x$?

- (1) $b^N = x$ (3) $x^b = N$
 (2) $N^b = x$ (4) $b^x = N$

26 What is the value of $\cos \left(\text{Arc tan } \frac{\sqrt{7}}{3} \right)$?

- (1) $\frac{3}{4}$ (3) $\frac{3\sqrt{7}}{7}$
 (2) $\frac{3}{16}$ (4) $\frac{\sqrt{7}}{4}$

27 What is the result of $R_{A,90^\circ} \circ R_{A,-180^\circ}$?

- (1)  (3) 
 (2)  (4) 

28 If a fair coin is tossed three times, the probability of obtaining exactly one tail is

- (1) $\frac{1}{8}$ (3) $\frac{3}{8}$
 (2) $\frac{2}{8}$ (4) $\frac{4}{8}$

29 The roots of the equation $2x^2 - 7x - 3 = 0$ are

- (1) rational (3) imaginary
 (2) irrational (4) equal

30 The expression $3 \sin \frac{1}{2}x$ reaches its maximum value when x , expressed in radians, equals

- (1) $\frac{\pi}{2}$ (3) 3
 (2) $\frac{3}{2}$ (4) π

31 The set of integers fails to meet the requirements for a field with respect to addition and multiplication because it lacks

- (1) multiplicative inverse
 (2) multiplicative identity
 (3) additive identity
 (4) closure

32 For all values of y , $y \neq 1$, the expression

$$\frac{6}{y-1} \cdot \frac{5-5y}{10} \text{ is equivalent to}$$

(1) $-\frac{1}{3}$

(3) -3

(2) $3(y-1)$

(4) $\frac{3}{5}(1-y)$

33 The graph of the sum of $(2 + 3i)$ and $(-1 - 6i)$ lies in Quadrant

(1) I

(3) III

(2) II

(4) IV

34 The solution of $|3x - 2| < 4$ is

(1) $-\frac{2}{3} < x < 2$

(3) $x > -\frac{2}{3}$

(2) $x < 2$

(4) $x < -\frac{2}{3}$ or $x > 2$

35 The value of $2i^8$ is equivalent to

(1) $2i$

(3) -2

(2) 2

(4) $-2i$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. All work, including calculations, must be shown on your answer paper. [40]

36 a Graph the equation $y = \cos \frac{1}{2}x$ for values of x in the interval $-\pi \leq x \leq \pi$. [4]

b On the same set of axes, sketch the transformation of the graph drawn in part a under $T_{\pi,0}$ and label it b . [4]

c If $\cos x = -\frac{14}{64}$ and x is in the second quadrant, find $\cos \frac{1}{2}x$ and express in simplest form. [2]

37 a Draw and label the graph of the equation $xy = 6$ in the interval $-6 \leq x \leq 6$. [4]

b On the same set of axes, draw and label the graph of the image of $xy = 6$ after a rotation of 90° . [2]

c Write the equation of the graph drawn in part b. [1]

d On the same set of axes, draw and label the graph of the image of $xy = 6$ after a dilation of 2. [2]

e Write the equation of the graph drawn in part d. [1]

38 a If A and B are acute angles such that $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, find, in simplest fractional form, the value of $\tan(A + B)$. [5]

b Using logarithms, find $(\sqrt[5]{1.91})^3$ to the nearest tenth. [5]

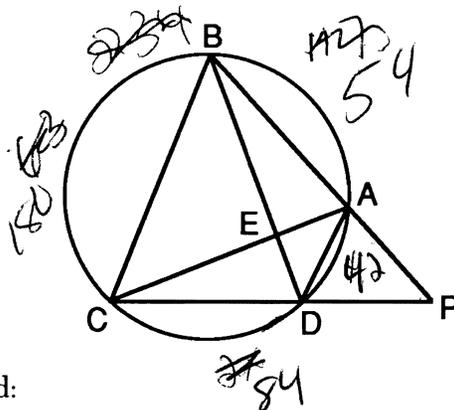
39 a Solve for x :

$$\frac{x}{x-2} - \frac{8}{x+3} = \frac{10}{x^2+x-6} \quad [6]$$

b Express the third term of $(1 + i)^7$ in simplest form. [4]

40 A frame is constructed in the form of an isosceles trapezoid. Each base angle measures $72^\circ 20'$, the longer base is 12.0 feet, and each of the nonparallel sides measures 5.0 feet. Find, to the nearest tenth, the number of feet in a diagonal brace of this frame. [5.5]

41 In the accompanying diagram, quadrilateral $ABCD$ is inscribed in the circle, diagonals \overline{AC} and \overline{BD} intersect at E , sides \overline{BA} and \overline{CD} are extended to P , $m\widehat{AD}:m\widehat{DC} = 1:2$, $m\widehat{ABC} = 234$, and $m\widehat{BC}$ is 54 less than twice $m\widehat{AB}$.



Find:

a $m\widehat{AB}$ [2]

b $m\angle ABD$ [2]

c $m\angle BPC$ [2]

d $m\angle AEB$ [2]

e $m\angle ADP$ [2]

42 Using the following set of score data, find the

a mean [2]

b standard deviation to the nearest tenth [6]

c probability that a randomly selected score falls within one standard deviation of the mean [2]

x_i	f_i
60	3
70	4
75	6
80	4
90	3