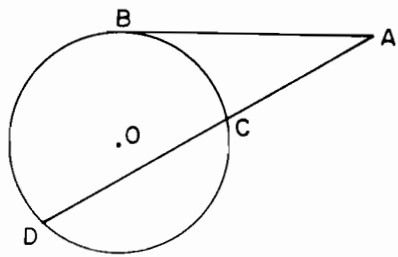
HIGH SCHOOL MATHEMATICS: COURSE III–JUNE 1982 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in terms of π or in radical form.

1. Express 105° in radian measure.
2. What are the coordinates of the image of point A $(-2, -5)$ after a reflection over the line $y = -1$?
3. Find the value of $\sum_{n=1}^{3} (n^2 + n)$.
4. If $\log_x 3 = \frac{1}{3}$, find x.
5. In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O. If $AC = 4$ and $CD = 5$, find AB.



6. Express $\cos 128^\circ$ as a function of a positive acute angle.
7. Express in *simplest form*: $\dfrac{\frac{1}{x^2} - 1}{1 + \frac{1}{x}}$
8. In a circle with radius 4.5 centimeters, find, in centimeters, the length of the arc intercepted by a central angle of 3 radians.
9. Find $\cos(\text{Arc tan } \sqrt{3})$.
10. If $f(x) = (3x)^{-2}$, find $f(2)$.
11. Express the sum of $(3 + \sqrt{-16})$ and $(7 - \sqrt{-81})$ in the form $a + bi$.
12. Combine and reduce to *lowest terms*: $\dfrac{4x}{2x + 6} + \dfrac{x}{x + 3}$
13. In triangle ABC, $m\angle C = 90$, side $c = 13$ centimeters, and side $a = 5$ centimeters. Find the area of the triangle in square centimeters.
14. Find the third term of the expansion of $(x - 2)^6$.
15. If $\log N = \log A + 3 \log B$, solve for N in terms of A and B.
16. Solve for x: $2^{3x+2} = 4^{2x}$

17. If $1 + i$ and $1 - i$ are the roots of the quadratic equation $x^2 + bx + c = 0$, what is the value of c?

18. In triangle ABC, $m\angle A = 40$, $m\angle B = 50$, and side $c = 80$ centimeters. Find side b to the *nearest centimeter*.

19. Express in *simplest form*: $\dfrac{9}{\sqrt{27}} + \sqrt{12}$

20. If $\sin x = -\frac{2}{3}$ and $\cos x > 0$, in which quadrant does angle x terminate?

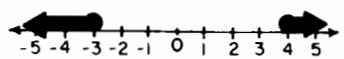
21. The probability of guessing the correct answer on a true-false question is $\frac{1}{2}$. If Mary guesses the answers to five true-false questions, what is the probability that she will get exactly one wrong answer?

Directions (22–35): For *each* question chosen, write on a separate sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

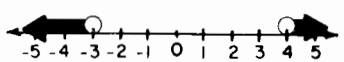
22. The expression $(\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ)$ is equal to
(1) 1 (2) $\frac{1}{2}$ (3) $\dfrac{\sqrt{3}}{2}$ (4) 0

23. Which is the graph of the solution set of $|1 - 2x| < 7$?

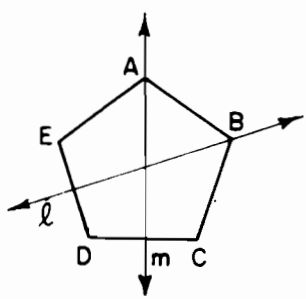
(1) 

(3) 

(2) 

(4) 

24. In the accompanying figure, ℓ and m are symmetry lines for regular pentagon $ABCDE$. What is $r_m \circ r_\ell(\overline{DC})$?
(1) $\overline{BA}$ (2) $\overline{AE}$ (3) $\overline{CB}$ (4) $\overline{DC}$



25. If $f(x) = 2\cos x + \sin 2x$, the numerical value of $f\left(\dfrac{\pi}{2}\right)$ is
(1) -1 (2) 2 (3) 0 (4) -2

26. Which geometric figure has 120° rotational symmetry?
(1) square (2) rhombus (3) regular pentagon (4) equilateral triangle

27. The domain of the relation $y = \dfrac{4}{\sqrt{x-1}}$ is the set
(1) $\{x|x > 1\}$ (2) $\{x|x \geq 1\}$ (3) $\{x|x < 1\}$ (4) $\{x|x > 2\}$

28. If the graph of $y = 2x^2 + 4x + c$ does *not* intersect the x-axis, then c may be equal to
(1) 1 (2) 2 (3) -2 (4) 5

29. In triangle ABC, $a = 2$, $b = 5$, and $\cos C = \frac{1}{2}$. The length of side c is
(1) $\sqrt{19}$ (2) $\sqrt{29}$ (3) $\sqrt{34}$ (4) $\sqrt{39}$

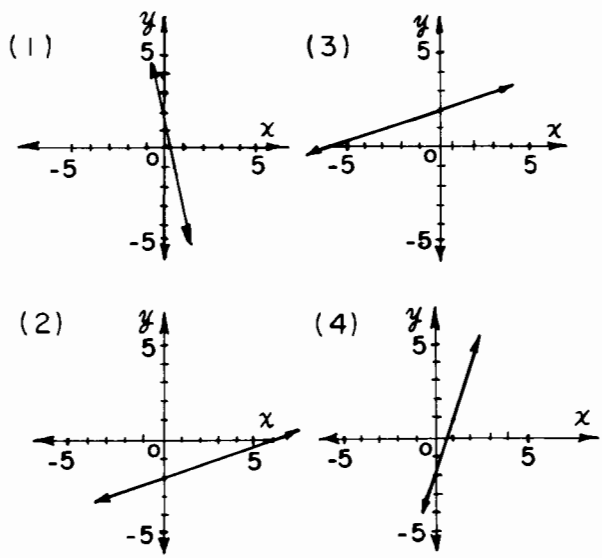
30. The graph of which equation has line symmetry with respect to the y-axis?
(1) $y = \sin x$ (2) $y = \tan x$ (3) $y = \cos x$ (4) $y = x$

31. The equation $y = \tan x$ is graphed in the interval $0 \leq x \leq \frac{\pi}{2}$ and is reflected over the x-axis.

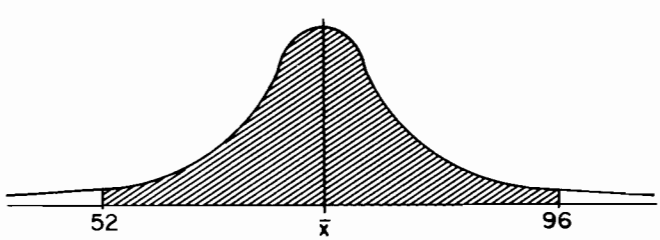
On this reflection, point $\left(\frac{\pi}{4}, y\right)$ has which value for y?

(1) 1 (2) -1 (3) 0 (4) $\frac{\sqrt{3}}{3}$

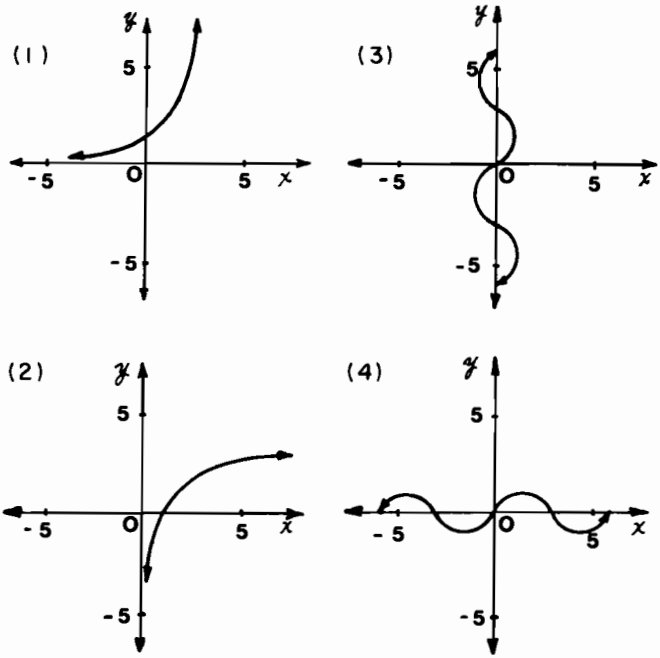
32. Which is the graph of the inverse of the function $y = 3x + 6$?



33. In the following diagram, the shaded area represents approximately 95% of the scores on a standardized test. If these scores ranged from 52 to 96, which could be the standard deviation?
(1) 11 (2) 22 (3) 76 (4) 44



34. Which graph does *not* represent a function?



35. The expression $\csc y + 1$ is equivalent to

(1) $\dfrac{\cot y}{\csc y - 1}$ (2) $\dfrac{\sin y + 1}{\sin y}$ (3) $\cot y$ (4) $\dfrac{1 + \cos y}{\cos y}$

Part II

Answer four questions from this part. Show all work unless otherwise directed.

36. *a.* On the same set of axes, sketch and label the graphs of $y = \tan x$ and $y = 2\cos x$ for the values of x in the interval $0 \leq x \leq 2\pi$. [8]
b. State the number of values of x in the interval $0 \leq x \leq 2\pi$ which satisfy the equation $\tan x = 2\cos x$. [2]

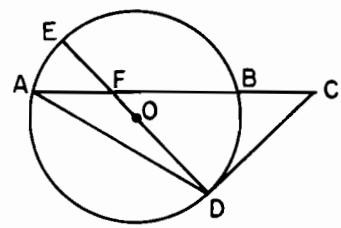
37. Triangle ABC has coordinates of $A(1, 1)$, $B(3, 1)$, and $C(1, 4)$.
a. Graph $\triangle ABC$ and graph and state the coordinates of its image $\triangle A'B'C'$ after the dilation D_3. [3]
b. Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after the reflection W: $(x, y) \rightarrow (x, -y)$. [3]
c. Graph and state the coordinates of $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after the glide reflection G: $(x, y) \rightarrow (y, x - 2)$. [3]
d. Which of the transformations above, D_3, W, or G, is *not* an isometry? [1]

38. *a*. Find to the *nearest degree* all values of θ in the interval $0° \leq \theta \leq 360°$ which satisfy the equation $\sec^2 \theta = 4 - 2 \tan \theta$. [6]
    b. Solve the equation $x^2 + 8x + 20 = 0$ and express its roots in the form $a + bi$. [4]

39. In triangle ABC, $b = 20$, $c = 12$, and $m\angle A = 28°50'$.
    a. Find the length of side a to the *nearest unit*. [6]
    b. Find the area of $\triangle ABC$ to the *nearest square unit*. [4]

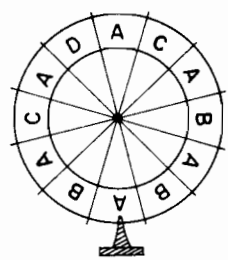
40. *a*. Sketch and label the graph of $y = 3^x$. [4]
    b. On the same set of axes, sketch the reflection of the graph of $y = 3^x$ over the line $y = x$. [4]
    c. Write the equation for the reflected graph in part *b*. [2]

41. In the accompanying diagram, $\overline{CD}$ is tangent to circle O, secant $\overline{ABC}$, chord $\overline{AB} \cong$ chord $\overline{AD}$, $m\overset{\frown}{AB} = 150$, and chord $\overline{EOD}$ is a diameter.
    Find:



    a. $m\overset{\frown}{AD}$ [2]
    b. $m\angle BAD$ [2]
    c. $m\angle ACD$ [2]
    d. $m\angle EFB$ [2]
    e. $m\angle EDA$ [2]

42. The wheel shown in the accompanying diagram is divided into twelve regions of equal area labeled as shown. On any spin of the wheel, each of the regions is equally likely to stop at the arrow.



    a. Find:
        (1) $P(A)$ [1]
        (2) $P(B)$ [1]
    b. Find the probability of:
        (1) getting *at least* three A's in four spins of the wheel [4]
        (2) getting *no more than* one B in three spins of the wheel [4]