The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 18, 1980 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

1 Express $72°$ in radian measure.

2 If $f(x) = x^{\frac{4}{3}}$, find $f(-27)$.

3 In a circle, a central angle of 3 radians intercepts an arc of 18 centimeters. What is the radius, in centimeters, of the circle?

4 Express in *simplest form*: $\dfrac{\frac{1}{2} + \frac{1}{x}}{\frac{1}{x}}$

5 Find the image of (1,5) when it is reflected over the line $y = x$.

6 Evaluate: $\sum_{k=5}^{7} (k-2)^2$

7 What is the amplitude of the graph of $y = \cos 2x$?

8 Chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E. If $AE = 4$, $EB = 5$, and $CE = 2$, find ED.

9 If $\sin x = \frac{3}{5}$, what is the value of $\cos 2x$?

10 If $\theta = \text{Arc}\cos\left(\frac{\sqrt{3}}{2}\right)$, what is the measure of angle θ?

11 If $\tan A = 0.4750$, find the value of A to the *nearest minute*.

12 A translation maps $P(4,-3)$ onto $P'(0,0)$. Find the coordinates of Q', the image of $Q(-2,1)$, under the same translation.

13 If $\tan A = \dfrac{-5}{12}$ and $\cos A > 0$, find $\sin A$.

14 Express $\sin(-170°)$ as a function of a positive acute angle.

15 Solve for x: $\dfrac{1}{15} + \dfrac{1}{10} = \dfrac{1}{x}$

16 What is the image of the point $(-3,-6)$ under a rotation of $90°$ counterclockwise about the origin?

17 If $\log_4 x = 3$, find x.

18 Find the value of $\cos \dfrac{5\pi}{3}$.

19 In triangle ABC, $\sin A = 0.8$, $\sin B = 0.3$, and $a = 24$. Find the length of side b.

Directions (20–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

20 The sum of $\sqrt{-2}$ and $\sqrt{-18}$ is

(1) $6i$ (3) $5i\sqrt{2}$

(2) $2i\sqrt{5}$ (4) $4i\sqrt{2}$

21 The solution set of $2^{x^2+3x} = 2^{-1}$ is

(1) $\{1\}$ (3) $\{1,-1\}$

(2) $\{-1\}$ (4) $\{\ \}$

22 The product of $(2-2i)$ and $(2+2i)$ is

(1) 0 (3) $4-4i$

(2) 8 (4) 4

23 If $x = \dfrac{a\sqrt{b}}{c}$, then $\log x$ is equal to

(1) $\log a + \frac{1}{2}\log b - \log c$

(2) $\log a + 2\log b - \log c$

(3) $\log a - \frac{1}{2}\log b + \log c$

(4) $\log a - 2\log b - \log c$

24 Which represents the solution set for x in the inequality $|2x-1| < 7$?

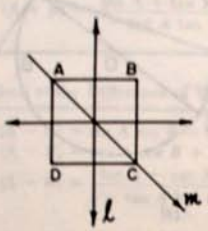
(1) $\{x|x < -3 \text{ or } x > 4\}$ (3) $\{x|-4 < x < 3\}$

(2) $\{x|x < -4 \text{ or } x > 3\}$ (4) $\{x|-3 < x < 4\}$

25 If $\sin (A - 30)^\circ = \cos 60^\circ$, the number of degrees in the measure of angle A is
(1) 30
(2) 60
(3) 90
(4) 120

26 Which kind of symmetry does a rhombus have?
(1) line symmetry, only
(2) point symmetry, only
(3) both line and point symmetry
(4) neither line nor point symmetry

27 In the accompanying figure, ℓ and m are symmetry lines. What is $r_\ell \circ r_m(\overline{AB})$?



(1) $\overline{AB}$
(2) $\overline{BC}$
(3) $\overline{CD}$
(4) $\overline{DA}$

28 The expression $(\cot \theta)(\sec \theta)$ is equivalent to
(1) $\csc \theta$
(2) $\sin \theta$
(3) $\cos \theta$
(4) $\tan \theta$

29 If $\sin \theta = \frac{1 - \sqrt{17}}{4}$, then angle θ lies in which quadrants?
(1) I and II, only
(2) II and IV, only
(3) III and IV, only
(4) I, II, III, and IV

30 A property *not* preserved under a line reflection is
(1) angle measure
(2) collinearity
(3) distance
(4) orientation

31 What is the domain of the function $f(x) = \sqrt{x - 2}$?
(1) $\{x | x \geq 0\}$
(2) $\{x | x \geq 2\}$
(3) $\{x | x \leq 2\}$
(4) $\{x | x \geq -2\}$

32 If the mean of a test score is 30 and the standard deviation is 3.7, which score could be expected to occur *less than* 5% of the time?
(1) 35
(2) 33.8
(3) 25
(4) 22

33 In the interval $0^\circ \leq \theta \leq 360^\circ$, how many values of θ satisfy the equation $3 \sin^2 \theta + \sin \theta - 2 = 0$?
(1) 1
(2) 2
(3) 3
(4) 4

34 In triangle ABC, $a = 2$, $b = 3$, and $c = 4$. What is the value of $\cos C$?
(1) $-\frac{1}{16}$
(2) $\frac{1}{16}$
(3) $-\frac{1}{4}$
(4) $\frac{1}{4}$

35 What is the third term in the expansion of $(a - 3b)^5$?
(1) $90a^3b^2$
(2) $45a^3b^2$
(3) $-45a^3b^2$
(4) $-90a^3b^2$

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549

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

36 *a* Find, to the *nearest degree*, all values of θ in the interval $0° \leq \theta \leq 360°$ which satisfy the equation $7 \cos \theta + 1 = 6 \sec \theta$. [6]

b For all values of θ for which the expressions are defined, prove the identity:

$$\tan \theta + \cot \theta = \sec \theta \csc \theta \quad [4]$$

37 *a* On the same set of axes, sketch the graphs of $y = 2 \sin x$ and $y = \cos \frac{1}{2}x$ as x varies from 0 to 2π radians. [8]

b State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $2 \sin x = \cos \frac{1}{2}x$. [2]

38 *a* Solve the equation $x^2 - 4x = -13$ and express the roots in the form $a + bi$. [6]

b Using logarithms, solve the equation $3^{2x} = 4$ for x to the *nearest tenth*. [4]

39 *a* Two consecutive sides of a parallelogram are 8 centimeters and 10 centimeters, respectively. If the length of the longer diagonal of the parallelogram is 14 centimeters, find the measure of the largest angle of the parallelogram to the *nearest degree*. [7]

b Using your answer to part *a*, find the area of the parallelogram to the *nearest square centimeter*. [3]

40 The ages of ten teachers at George Washington Elementary School are 33, 23, 36, 29, 36, 36, 33, 29, 36, and 29. Determine the standard deviation of these ages to the *nearest tenth*. [10]

41 Given: F is the transformation $(x,y) \rightarrow (-y,-x)$
U is the transformation $(x,y) \rightarrow (x-2,y+4)$
N is the transformation $(x,y) \rightarrow (2x,2y)$

The coordinates of $\triangle ABC$ are $A(1,2)$, $B(4,0)$, and $C(3,-2)$.

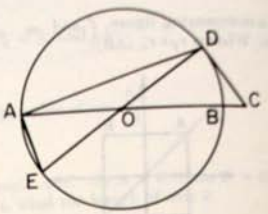
a Sketch $\triangle ABC$ and its image $\triangle A'B'C'$ after the transformation F. [3]

b Sketch $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after the transformation U. [3]

c Sketch $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after the transformation N. [3]

d Which transformation, F, U, or N, is a dilation? [1]

42 In circle O, diameter $\overline{AB}$ is extended to point C. $\overline{CD}$ is tangent to the circle at D, $\overline{DE}$ is a diameter, and $m\overset{\frown}{BD} : m\overset{\frown}{AD} = 1:4$.



Find:

a $m\overset{\frown}{BD}$ [2]

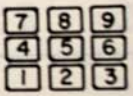
b $m\angle E$ [2]

c $m\angle C$ [2]

d $m\overset{\frown}{AE}$ [2]

e $m\angle ADE$ [2]

43 The numeric key pad on a calculator is arranged as shown in the diagram below. The probability of pressing any key at random is the same for each key.



a Find:

(1) $P(6)$ [1]

(2) $P(\text{even number})$ [1]

(3) $P(\text{odd number})$ [1]

b Find the probability of:

(1) pressing exactly 2 even numbers on three random presses [3]

(2) getting at least 2 even numbers on three random presses [4]

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\bar{x} - x_i)^2}$$

[OVER]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 18, 1980 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] For questions 20–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $\frac{2\pi}{5}$

(2) 9

(3) 6

(4) $\frac{x+2}{2}$

(5) (5,1)

(6) 50

(7) 1

(8) 10

(9) $\frac{7}{25}$

(10) 30 *or* $\frac{\pi}{6}$

(11) 25° 24′

(12) (−6,4)

(13) $-\frac{5}{13}$

(14) $-\sin 10°$ *or* $-\cos 80°$

(15) 6

(16) (6,−3)

(17) 64

(18) $\frac{1}{2}$ *or* 0.5

(19) 9

(20) 4

(21) 2

(22) 2

(23) 1

(24) 4

(25) 2

(26) 3

(27) 2

(28) 1

(29) 3

(30) 4

(31) 2

(32) 4

(33) 3

(34) 3

(35) 1

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* 31, 180, 329 [6]

(37) *b* 3 [2]

(38) *a* $2 \pm 3i$ [6]
b 0.6 [4]

(39) *a* 102 [7]
b 78 [3]

(40) 4.2 [10]

(41) *d* *N* [1]

(42) *a* 36 [2]
b 72 [2]
c 54 [2]
d 36 [2]
e 18 [2]

(43) *a* (1) $\frac{1}{9}$ [1]

(2) $\frac{4}{9}$ [1]

(3) $\frac{5}{9}$ [1]

b (1) $\frac{240}{729}$ [3]

(2) $\frac{304}{729}$ [4]