The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, January 27, 1999 — 9:15 a.m. to 12:15 p.m., only

Notice ...

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1 \qquad \tan A = \frac{\sin A}{\cos A}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\cot^2 A + 1 = \csc^2 A \qquad \cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2} A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2} ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.   [60]

1 In the accompanying diagram of circle O, $\overrightarrow{XA}$ and $\overrightarrow{XB}$ are tangents and $m\angle XAB = 75$. Find $m\angle X$.
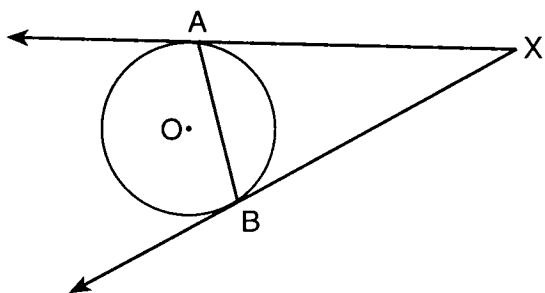
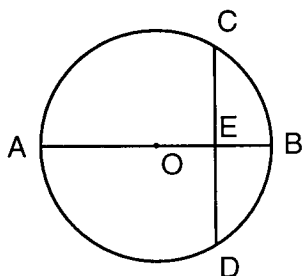2 Translation T maps point $(2,6)$ to point $(4,-1)$. What is the image of point $(-1,3)$ under translation T?

3 Express the sum of $2\sqrt{-49}$ and $-3\sqrt{-16}$ as a monomial in terms of i.

4 If $f(x) = x^2 - 3x$, find $f(-1.8)$.

5 If $f(x) = \sin 3x + \cos x$, what is $f\left(\frac{\pi}{2}\right)$?
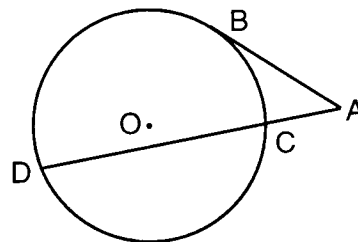
6 Evaluate: $\displaystyle\sum_{k=1}^{3}(k+1)^2$

7 In the accompanying diagram of circle O, diameter $\overline{AB}$ is perpendicular to chord $\overline{CD}$ at E, $CD = 8$, and $EB = 2$. What is the length of the diameter of circle O?

8 Express $75°$ in radian measure.

9 In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O from point A. If $AC = 4$ and $CD = 12$, find AB.

10 Express in simplest form: $\dfrac{\frac{3}{4}+\frac{3}{x}}{\frac{1}{x}+\frac{1}{4}}$

11 In $\triangle ABC$, $m\angle A = 33$, $a = 12$, and $b = 15$. Find $\sin B$ to the *nearest thousandth*.

12 Factor completely: $x^4 - 16$

13 In $\triangle ABC$, $a = 2$, $c = 6$, and $\cos B = \frac{1}{6}$. Find b.

14 The width of a rectangle with constant area varies inversely as its length. If the width is 4 when the length is 12, find the width when the length is 16.

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 For which value(s) of x is the function $f(x) = \dfrac{x^2-9}{x-7}$ undefined?

(1) 9   (3) 3, only

(2) 3 and -3 (4) 7

16 The solution set of $2^{x+1} = 8$ is

(1) { }
(3) {3}
(2) {2}
(4) {4}

17 The expression $\cos 70° \cos 10° + \sin 70° \sin 10°$ is equivalent to

(1) $\cos 60°$
(3) $\sin 60°$
(2) $\cos 80°$
(4) $\sin 80°$

18 If the image of A after a dilation of -2 is $A'(-8,6)$, what are the coordinates of A?

(1) $(4,-3)$
(3) $(16,-12)$
(2) $(-4,3)$
(4) $(-16,12)$

19 If θ is an angle in Quadrant I and $\tan^2 \theta - 4 = 0$, what is the value of θ to the *nearest degree?*

(1) 1
(3) 63
(2) 2
(4) 75

20 If $\log_4 x = 3$, then x is equal to

(1) 7
(3) 64
(2) 12
(4) 81

21 The value of $\sin (\text{Arc} \cos 1)$ is

(1) 1
(3) $\frac{1}{2}\sqrt{3}$
(2) $\frac{1}{2}$
(4) 0

22 If $\cos A > 0$ and $\csc A < 0$, in which quadrant does the terminal side of $\angle A$ lie?

(1) I
(3) III
(2) II
(4) IV

23 If -1 and 7 are the roots of the quadratic equation $x^2 + kx - 7 = 0$, then k must be

(1) -7
(3) 6
(2) -6
(4) 8

24 The expression $\sec x \sin 2x$ is equivalent to

(1) $\frac{1}{2}$
(3) $2 \cos x$
(2) 2
(4) $2 \sin x$

25 A fair die is tossed five times. What is the probability of obtaining exactly three 4's?

(1) $\frac{250}{7776}$
(3) $\frac{1250}{7776}$
(2) $\frac{10}{7776}$
(4) $\frac{90}{1024}$

26 If $|2x + 3| < 1$, then the solution set contains

(1) only negative real numbers
(2) only positive real numbers
(3) both positive and negative real numbers
(4) no real numbers

27 Which relation is a function?

(1) $y = \cos x$
(3) $x = y^2$
(2) $x = 4$
(4) $x^2 + y^2 = 16$

28 The roots of the equation $x^2 + 4x + 2 = 0$ are

(1) real, rational, and equal
(2) real, rational, and unequal
(3) real, irrational, and unequal
(4) imaginary

29 In the accompanying diagram of a unit circle, $\overline{BA}$ is tangent to circle O at A, $\overline{CD}$ is perpendicular to the x-axis, and $\overline{OC}$ is a radius.
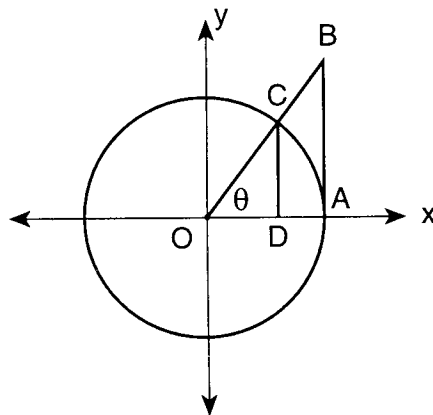
Which distance represents $\sin \theta$?

(1) OD
(3) BA
(2) CD
(4) OB

30 A standardized test with a normal distribution of scores has a mean score of 43 and a standard deviation of 6.3. Which range would contain the score of a student in the 90th percentile?

(1) 30.4–36.7
(2) 36.7–43.0
(3) 43.0–49.3
(4) 49.3–55.6

31 The graph of the equation $xy = 5$ forms

(1) an ellipse
(2) a hyperbola
(3) a line
(4) a parabola

32 The value of $(1 - i)^2$ is

(1) 0
(2) 2
(3) $-2i$
(4) $2 - 2i$

33 The solution set of the equation $\sqrt{2x+15} = x$ is

(1) {5,–3}
(2) {5}
(3) {–3}
(4) { }

34 What are the coordinates of the image of $P(-2,5)$ after a clockwise rotation of 90° about the origin?

(1) (–5,–2)
(2) (–2,–5)
(3) (2,5)
(4) (5,2)

35 What is the best approximation of the standard deviation of the measures –4, –3, 0, 8, and 9?

(1) 1
(2) 2
(3) 5
(4) 10

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = -\sin x$ and $y = 2 \cos x$ in the interval $0 \le x \le 2\pi$. [8]

 b Using the graphs sketched in part *a*, determine the number of solutions to the equation $2 \cos x = -\sin x$ in the interval $0 \le x \le 2\pi$. [2]

37 In the accompanying diagram of circle O, $m\overparen{AC} = 140$, $m\overparen{AE} = 130$, $m\overparen{AB}{:}m\overparen{BC} = 6{:}4$, $\overline{PD}$ is a tangent, secant $\overline{PCE}$ intersects diameter $\overline{AD}$ at F, and secant $\overline{PBA}$ is drawn.
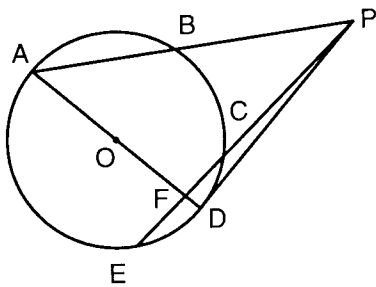
Find:

a $m\overparen{ED}$ [2]

b $m\overparen{AB}$ [2]

c $m\angle BAD$ [2]

d $m\angle APE$ [2]

e $m\angle EFD$ [2]

38 *a* Express in simplest form:

$$\frac{2x - 8}{x^2 + x - 12} \div \frac{20 - 5x}{2x^2 - 5x - 3}$$ [4]

 b Prove the following identity:

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \cot \theta \sec \theta$$ [6]

39 *a* On graph paper, sketch the graph of the equation $y = 3^x$ in the interval $-2 \le x \le 2$. [2]

 b On the same set of axes, sketch the graph of the equation $y = 6$. [1]

 c Based on the graphs sketched in parts *a* and *b*, between which two consecutive integers does the solution of $3^x = 6$ lie? Explain your answer. [1,2]

 d Find x to the *nearest hundredth*: $3^x = 6$ [4]

40 In $\triangle ABC$, $AC = 8$, $BC = 17$, and $AB = 20$.

 a Find the measure of the largest angle to the *nearest degree*. [6]

 b Find the area of $\triangle ABC$ to the *nearest integer*. [4]

41 *a* A mathematics quiz has five multiple-choice questions. There are four possible responses for each question. Jennifer selects her responses at random on every question.

 (1) What is the probability she will select the correct response for *at most* one question? [3]

 (2) What is the probability she will select the correct response to *at least* three questions? [4]

 b Find, in simplest form, the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^6$. [3]

42 Find, to the *nearest degree*, all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $2 \sin^2 \theta + 2 \cos \theta - 1 = 0$. [10]

SEQUENTIAL MATH – COURSE III

Wednesday, January 27, 1999 — 9:15 a.m. to 12:15 p.m., only

Part I Score	. . . .
Part II Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Pupil . . . .   Sex: ☐ Male ☐ Female   Grade . . . .

Teacher . . . . School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

Tear Here

FOR TEACHERS ONLY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, January 27, 1999 — 9:15 a.m. to 12:15 p.m., only

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 30	(11) 0.681	(21) 4	(31) 2
(2) (1,–4)	(12) $(x^2 + 4)(x + 2)(x - 2)$	(22) 4	(32) 3
(3) $2i$	(13) 6	(23) 2	(33) 2
(4) 8.64	(14) 3	(24) 4	(34) 4
(5) –1	(15) 4	(25) 1	(35) 3
(6) 29	(16) 2	(26) 1	
(7) 10	(17) 1	(27) 1	
(8) $\frac{5\pi}{12}$	(18) 1	(28) 3	
(9) 8	(19) 3	(29) 2	
(10) 3	(20) 3	(30) 4	

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *b* 2   [2]

(37) *a* 50   [2]
     b 84   [2]
     c 48   [2]
     d 37   [2]
     e 95   [2]

(38) *a* $-\dfrac{2(2x+1)}{5(x+4)}$ [4]

(39) *c* 1 and 2   [1]
     The graph of $y = 6$ intersects the graph of $y = 3^x$ for some value of x between 1 and 2.
          or
     An equivalent response   [2]

     d 1.63   [4]

(40) *a* 100   [6]
     b 67   [4]

(41) *a* (1) $\dfrac{648}{1024}$ [3]

        (2) $\dfrac{106}{1024}$ [4]

     b $20x^3$ [3]

(42) 111 and 249   [10]