The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, January 27, 1998 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$S.D. = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.   [60]

1 Express the sum of $\sqrt{-25}$ and $2\sqrt{-9}$ as a monomial in terms of i.

2 In $\triangle ABC$, $b = 6$, $c = 3$, and $\sin B = 0.4$. Find the value of $\sin C$.

3 An angle that measures $\frac{7\pi}{4}$ radians is in standard position. In which quadrant does its terminal side lie?

4 In the accompanying diagram, segments $\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ are tangent to circle O at A, B, and C, respectively. If $SB = 3$, $BT = 5$, and $TR = 13$, what is the measure of $\overline{RS}$?
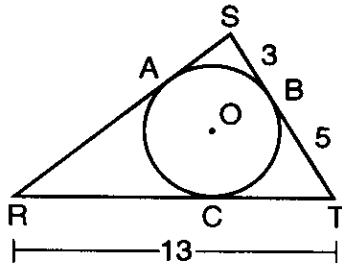
5 Solve for x: $\sqrt{5x + 2} - 3 = 0$

6 Evaluate: $\displaystyle\sum_{k=1}^{4}(k + 2)^3$

7 If $f(x) = (16x)^0 + x^{\frac{2}{3}}$, find $f(64)$.

8 Express in simplest form: $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{1}{y} + \dfrac{1}{x}}$

9 Find the image of $A(4,-2)$ under the transformation $r_{y=x}$.

10 Solve for y: $2^{(y-3)} = \dfrac{1}{16}$

11 If a fair coin is tossed five times, what is the probability of tossing *exactly* three heads?

12 If $f(x) = 2\sin^2 x + \sin x + 1$, find the value of $f\left(\dfrac{\pi}{6}\right)$.

13 Find $m\angle\theta$ in the interval $180° \leq \theta \leq 270°$ that satisfies the equation $2\cos\theta + 1 = 0$.

14 In the accompanying table, y varies inversely as x. What is the value of m?

x	2	4	m
y	18	9	3

15 In the accompanying diagram, tangent $\overline{CD}$ and secant $\overline{CBA}$ are drawn to circle O from external point C. If $DC = 4$ and $AB = 6$, find the length of $\overline{BC}$.
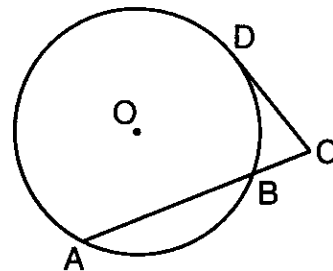
16 Write a single translation that is equivalent to $T_{3,-1}$ followed by $T_{-5,5}$.

17 Express the product of $4 - 3i$ and $2 + i$ in simplest $a + bi$ form.

18 If $\log_x \frac{1}{4} = -2$, find x.

Directions (19–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

19 Sin 50° cos 30° + cos 50° sin 30° is equivalent to
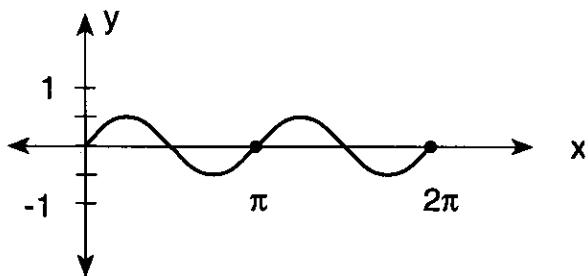(1) cos 80°   (3) cos 20°
(2) sin 20°   (4) sin 80°

20 If $f(x) = x - 3$ and $g(x) = x^3$, then $f(g(3))$ is
(1) 0   (3) 24
(2) 6   (4) 30

21 Which curve has only one line of symmetry?
(1) a circle   (3) a parabola
(2) an ellipse   (4) a hyperbola

22 Which equation is represented in the accompanying graph?

(1) $y = 2 \sin 2x$ (3) $y = 2 \sin \frac{1}{2}x$
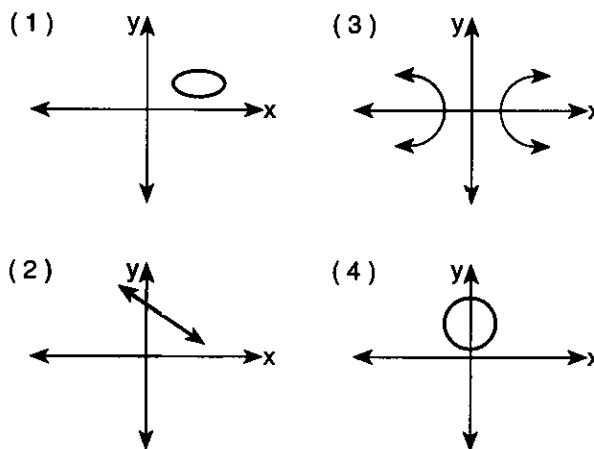(2) $y = \frac{1}{2} \sin \frac{1}{2}x$ (4) $y = \frac{1}{2} \sin 2x$

23 The solution set of $|x - 3| > 5$ is
(1) {$x < 8$ and $x < -2$}   (3) {$x < 8$ and $x > -2$}
(2) {$x < 8$ or $x < -2$}   (4) {$x > 8$ or $x < -2$}

24 The graph of the equation $y = 10^x$ lies entirely in Quadrants
(1) I and II   (3) I and IV
(2) II and III   (4) III and IV

25 Which graph of a relation is also a function?

26 If $\theta = $ Arc cos $\frac{\sqrt{2}}{2}$, what is the value of tan θ?
(1) 1   (3) $\frac{1}{\sqrt{2}}$
(2) $\sqrt{2}$ (4) $\frac{1}{2}$

27 If tan $\theta < 0$ and csc $\theta > 0$, in which quadrant does θ terminate?
(1) I   (3) III
(2) II   (4) IV

28 In the accompanying diagram of a unit circle, the ordered pair (x,y) represents the point where the terminal side of θ intersects the unit circle.
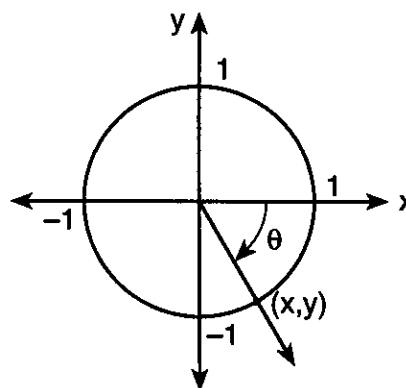
If $\theta = -\frac{\pi}{3}$, what is the value of y?
(1) $-\frac{\sqrt{3}}{2}$ (3) $-\sqrt{3}$
(2) $-\frac{\sqrt{2}}{2}$ (4) $-\frac{1}{2}$

29 What is the fourth term in the expansion of $(x - 2y)^5$?

(1) $80x^2y^3$ (3) $-80x^2y^3$

(2) $80xy^4$ (4) $-80xy^4$

30 The heights of a group of 1000 women are normally distributed. The mean height of the group is 170 centimeters (cm) with a standard deviation of 10 cm. What is the best approximation of the number of women between 170 cm and 180 cm tall?

(1) 950   (3) 340

(2) 680   (4) 170

31 For which value of x is $f(x)$ undefined if $f(x) = \dfrac{\cos x}{1 - \cos 2x}$?

(1) 1   (3) $\frac{\pi}{2}$

(2) $\frac{1}{2}$ (4) π

32 The graph of the equation $xy = -8$ is

(1) an ellipse   (3) a circle

(2) a hyperbola   (4) a parabola

33 The solution of $x^2 - 3x < 0$ is

(1) $0 < x < 3$ (3) $x < 0$ or $x > 3$

(2) $x > 3$ (4) $x < 0$

34 If $m\angle A = 45$, $AB = 10$, and $BC = 8$, the greatest number of distinct triangles that can be constructed is

(1) 1   (3) 3

(2) 2   (4) 0

35 In $\triangle ABC$, $a = 8$, $b = 9$, and $m\angle C = 135$. What is the area of $\triangle ABC$?

(1) 18   (3) $18\sqrt{2}$

(2) 36   (4) $36\sqrt{2}$

**Answers to the following questions are to be written on paper provided by the school.**

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.   [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = \cos 2x$ and $y = -2 \sin x$ in the interval $0 \le x \le 2\pi$.   [8]

   b Using the graphs sketched in part *a*, determine the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $-2 \sin x - \cos 2x = 3$.   [2]

37 Find all positive values of θ less than 360° that satisfy the equation $2 \cos 2\theta - 3 \sin \theta = 1$. Express your answers to the *nearest ten minutes* or *nearest tenth of a degree*.   [10]

38 *a* Given: $r = 2 - i$ and $s = 4 + 3i$

   (1) On graph paper, draw and label the graphs of these complex numbers.   [2]

   (2) On the same set of axes, graph the sum of r and s as drawn in part *a*(1) and label it *t*.   [1]

   (3) On the same set of axes, draw the image of *t* after a counterclockwise rotation of 90° and label it *t'*.   [2]

   b Express the roots of the equation $2x + \dfrac{5}{x} = 2$ in simplest $a + bi$ form.   [5]

GO RIGHT ON TO THE NEXT PAGE. ⟹

39 In the accompanying diagram of circle O, chord $\overline{AB}$ is parallel to diameter $\overline{EC}$, secant $\overline{PBD}$ intersects $\overline{EC}$ at F, tangent $\overline{PA}$ is drawn, $m\overarc{AB} = m\overarc{BC}$, and $m\overarc{CD} = 80$.
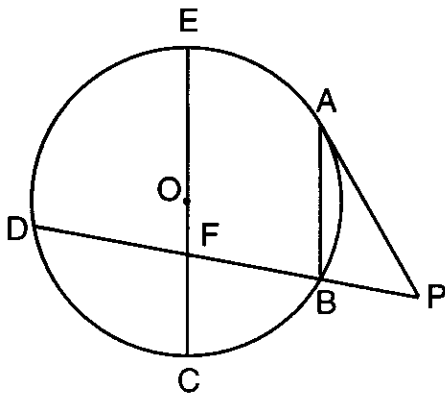
Find:

a $m\overarc{AE}$ [2]

b $m\angle ABD$ [2]

c $m\angle DFC$ [2]

d $m\angle P$ [2]

e $m\angle PAB$ [2]

40 a Solve for all values of x:

$$\frac{2x}{x+3} + \frac{3}{x-3} = \frac{8}{x^2-9} \qquad [5]$$

b If $\log_b R = 0.75$ and $\log_b S = 0.25$, find the value of

(1) $\log_b R^2 S$ [2]

(2) $\log_b \dfrac{\sqrt[3]{R}}{RS}$ [3]

41 In $\triangle ABC$, $a = 6$, $b = 7$, and $c = 10$.

a Find the measure of angle A to the *nearest ten minutes* or *nearest tenth of a degree*. [6]

b Using the result from part a, find the area of $\triangle ABC$ to the *nearest tenth*. [4]

42 The table below shows the scores on a writing test in an English class.

x_i	f_i
95	4
85	13
75	11
70	6
65	2

a Using the accompanying set of data, find both the mean and the standard deviation to the *nearest tenth*. [4]

b What is the number of scores that fall within one standard deviation of the mean ($\bar{x} \pm 1\sigma$)? [2]

c Find, to the *nearest tenth*, the percentage of scores in this set of data that are within one standard deviation of the mean. [1]

d What is the number of scores that fall within two standard deviations of the mean ($\bar{x} \pm 2\sigma$)? [2]

e Find the percentage of scores in this set of data that are within two standard deviations of the mean. [1]

SEQUENTIAL MATH – COURSE III

Tuesday, January 27, 1998 — 1:15 to 4:15 p.m., only

Part I Score	. . . .
Part II Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Pupil . . . .   Sex: ☐ Male ☐ Female   Grade . . . .

Teacher . . . .   School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

FOR TEACHERS ONLY

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 19–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $11i$	(11) $\frac{10}{32}$	(21) 3	(31) 4
(2) 0.2	(12) 2	(22) 4	(32) 2
(3) IV	(13) 240	(23) 4	(33) 1
(4) 11	(14) 12	(24) 1	(34) 2
(5) $\frac{7}{5}$	(15) 2	(25) 2	(35) 3
(6) 432	(16) $T_{-2,4}$	(26) 1	
(7) 17	(17) $11 - 2i$	(27) 2	
(8) $x - y$	(18) 2	(28) 1	
(9) $(-2,4)$	(19) 4	(29) 3	
(10) -1	(20) 3	(30) 3	

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *b* 1   [2]

(37) 14.5, 165.5, 270
$$or$$ [10]
14°30', 165°30', 270°

(38) *b* $\frac{1}{2} \pm \frac{3}{2}i$ [5]

(39) *a* 60   [2]
    b 80   [2]
    c 100   [2]
    d 50   [2]
    e 30   [2]

(40) *a* $\frac{1}{2}$, 1   [5]
    b (1) 1.75   [2]
       (2) −0.75   [3]

(41) *a* 36.2 *or* 36°10'   [6]
    b 20.7   [4]

(42) *a* $\bar{x}$ = 79.4, σ = 8.4   [4]
    b 24   [2]
    c 66.7   [1]
    d 36   [2]
    e 100   [1]