The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, January 28, 1997 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

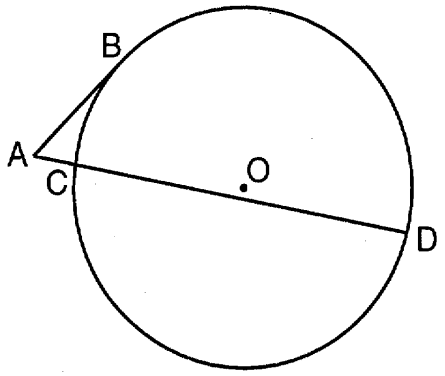
Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 In the accompanying diagram, $\overline{AB}$ is tangent to circle O at B and $\overline{ACD}$ is a secant. If $AB = 9$ and $AD = 27$, find AC.

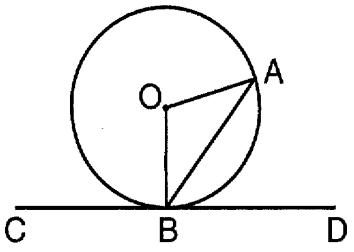


2 In terms of i, express in simplest form:

$$\sqrt{-64} - 3\sqrt{-4}$$

3 In $\triangle ABC$, $\sin A = \frac{2}{3}$, $\sin B = \frac{4}{5}$, and side $a = 20$. Find side b.

4 In the accompanying diagram, $\overline{CD}$ is tangent to circle O at B, AO and BO are radii, and chord $\overline{AB}$ is drawn. If $m\angle AOB = 108$, find $m\angle ABD$.



5 Solve for y: $\frac{4}{5y - 3} = \frac{2}{3y + 4}$

6 Solve for y: $3^{y+1} = 9^{y-1}$

7 Express 1.2π radians in degrees.

8 In which quadrant does the graph of the sum of $(-3 - 5i)$ and $(2 + 4i)$ lie?

9 Evaluate: $\sum_{n=1}^{3}(2n - 1)$

10 If $f(x) = x - 3$ and $g(x) = x^2$, what is the value of $(f \circ g)(2)$?

11 In $\triangle ABC$, $a = 8$, $b = 9$, and $\cos C = \frac{2}{3}$. Find c.

12 In a normal distribution, 68% of the scores fall between 72 and 86 and the mean is 79. What is the standard deviation?

13 In a circle with a radius of 4 centimeters, what is the number of radians in the central angle that intercepts an arc of 8 centimeters?

14 If x varies inversely as y, and $x = 9$ when $y = 8$, find x when $y = 12$.

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 If $f(x) = 3^x$, then $f(-2)$ equals

(1) $\frac{1}{9}$ (3) -6

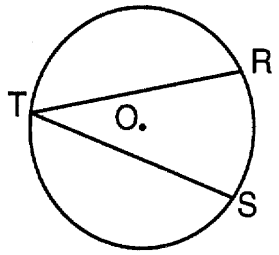
(2) 9 (4) -9

16 The expression $\frac{\cot \theta}{\csc \theta}$ is equivalent to

(1) $\frac{\cos \theta}{\sin^2 \theta}$ (3) $\tan \theta$

(2) $\sin \theta$ (4) $\cos \theta$

17 In the accompanying diagram of circle O, the measure of $\overset{\frown}{RS}$ is 64°.



What is m$\angle RTS$?

(1) 32
(2) 64
(3) 96
(4) 128

18 If $\sin A > 0$ and $(\sin A)(\cos A) < 0$, in which quadrant does $\angle A$ terminate?

(1) I
(2) II
(3) III
(4) IV

19 The solution set of the inequality $|x - 3| < 5$ is

(1) $\{x < 8$ and $x < -2\}$
(2) $\{x < 8$ or $x < -2\}$
(3) $\{x < 8$ and $x > -2\}$
(4) $\{x > 8$ or $x < -2\}$

20 For which value of x is $f(x) = \frac{\sin x}{\cos x}$ undefined?

(1) 0
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{2}$
(4) π

21 The expression $\cos 80° \cos 20° - \sin 80° \sin 20°$ is equivalent to

(1) $\cos 60°$
(2) $\cos 100°$
(3) $\sin 100°$
(4) $\sin 60°$

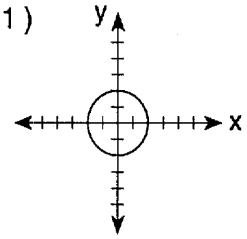
22 Which statement is true about the roots of the equation $\sqrt{x^2 - 5x + 5} = 1$?

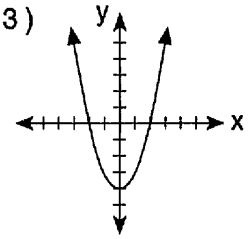
(1) The only root is 1.
(2) The only root is 4.
(3) Both 1 and 4 are roots.
(4) Neither 1 nor 4 is a root.

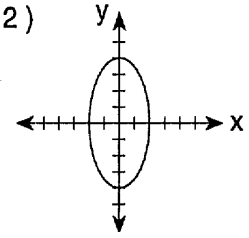
23 If the coordinates of P are $(-2,7)$, what are the coordinates of $(D_2 \circ r_{y=x})(P)$?

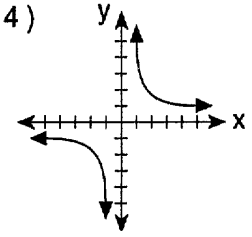
(1) (4,–14)
(2) (–14,4)
(3) (–4,14)
(4) (14,–4)

24 Which graph represents the equation $\frac{x^2}{4} + \frac{y^2}{4} = 1$?

(1)


(3)


(2)


(4)


25 What is the period of the graph of the equation $y = a \sin bx$?

(1) $\frac{2\pi}{a}$
(2) $\frac{2\pi}{b}$
(3) a
(4) b

26 What is the product of the roots of the equation $2x^2 - x - 2 = 0$?

(1) 1
(2) 2
(3) –1
(4) –2

27 What is the image of $A(5,2)$ under $R_{90°}$?

(1) (–5,2)
(2) (5,–2)
(3) (2,5)
(4) (–2,5)

28 The set $\{0,1,-1\}$ is closed under the operation of

(1) addition
(2) multiplication
(3) subtraction
(4) division

29 If $\sin (x + 20°) = \cos x$, the value of x is

(1) 35°
(2) 45°
(3) 55°
(4) 70°

30 The probability of Gordon's team winning any given game in a 5-game series is 0.3. What is the probability that Gordon's team will win *exactly* 2 games in the series?

(1) $(0.3)^2(0.7)^3$
(2) $5(0.3)^3(0.7)^2$
(3) $10(0.3)^2(0.7)^3$
(4) $5(0.3)^2(0.7)$

31 What is the solution set of the inequality $x^2 - 3x - 10 > 0$?

(1) $\{x \mid -2 < x < 5\}$
(2) $\{x \mid -5 < x < 2\}$
(3) $\{x \mid x < -5 \text{ or } x > 2\}$
(4) $\{x \mid x < -2 \text{ or } x > 5\}$

32 What is the middle term in the expansion of $(3x - 2y)^4$?

(1) $-6x^2y^2$
(2) $36x^2y^2$
(3) $-216x^2y^2$
(4) $216x^2y^2$

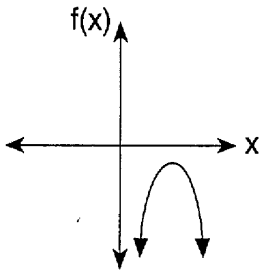
33 If the graphs of the equations $y = \log_3 x$ and $y = 2$ are drawn on the same set of axes, they will intersect where x is equal to

(1) 1
(2) 2
(3) 3
(4) 9

34 An obtuse angle of a parallelogram has a measure of 150°. If the sides of the parallelogram measure 10 and 12 centimeters, what is the area of the parallelogram?

(1) 30 cm^2
(2) 60 cm^2
(3) $60\sqrt{2}$ cm^2
(4) $60\sqrt{3}$ cm^2

35 The accompanying diagram shows a sketch of a quadratic function, $f(x)$.



What is the nature of the roots of the quadratic equation $f(x) = 0$?

(1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real, irrational, and unequal

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* Solve for x and express the roots in terms of i:

$$\frac{x+3}{3} + \frac{x+3}{x} = 2 \quad [4]$$

b Solve for x and express the roots in simplest $a + bi$ form:

$$x^2 = 6x - 10 \quad [6]$$

37 *a* Find, to the *nearest degree*, all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $3\sin^2\theta - \sin\theta - 2 = 0$. [8]

b Solve for x to the *nearest tenth*:

$$5^x = 30 \quad [2]$$

38 *a* On the same set of axes, sketch and label the graphs of the equations $y = -2\sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [8]

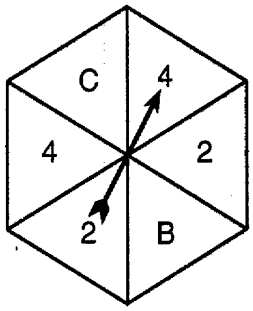
b Using the graphs sketched in part *a*, determine the number of points in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $\cos 2x = -2\sin x$. [2]

39 *a* The table below shows the set of score data for an English examination.

x_i	f_i
100	2
90	3
80	6
70	5
60	4

Find the standard deviation of these scores to the *nearest tenth*. [4]

b In the accompanying diagram, a regular hexagon with a spinner is divided into six equal areas labeled with a letter or a number.



If the spinner is spun four times, find the probability that it will land in a

(1) numbered area *at most* one time [3]

(2) lettered area *at least* three times [3]

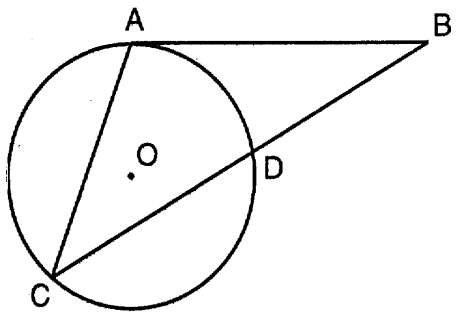
40 *a* Express in simplest form:

$$\frac{1 - \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{x}} \quad [5]$$

b For all values of x for which the expressions are defined, prove the following is an identity:

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x \quad [5]$$

41 In the accompanying diagram of circle O, $\triangle ABC$ is formed by tangent $\overline{AB}$, secant $\overline{BDC}$, and chord $\overline{AC}$; $\overline{CA} \cong \overline{CD}$; $m\overset{\frown}{AC} = 140$; and $AC = 10$.



Find:

a $m\overset{\frown}{AD}$ [2]

b $m\angle B$ [2]

c AB to the *nearest tenth* [6]

42 *a* *On your answer paper*, copy and complete the table for the values of y for the equation $y = \log_2 x$. [4]

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y					

b *On graph paper*, using the completed table, draw the graph of the equation $y = \log_2 x$ for the interval $\frac{1}{4} \leq x \leq 4$. Label the graph b. [2]

c On the same set of axes, reflect the graph drawn in part *b* in the y-axis and label it c. [2]

d On the same set of axes, reflect the graph drawn in part *b* in the line $y = x$ and label it d. [2]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Tuesday, January 28, 1997 — 9:15 a.m. to 12:15 p.m., only

Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Pupil ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 .....
2	12	22	32 .....
3	13	23	33 .....
4	14	24	34 .....
5	15	25	35 .....
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

THE STATE EDUCATION DEPARTMENT/THE UNIVERSITY OF THE STATE OF NEW YORK/ALBANY, N.Y. 12234

OFFICE OF STATE ASSESSMENT
ROOM 760 EBA
(518) 474-5900

January 30, 1997

TO: Principals of Public and Nonpublic High Schools

FROM: Annette C. Argyros
Office of State Assessment

SUBJECT: January 1997 Regents Examination in Earth Science
January 1997 Regents Examination in Sequential Mathematics, Course III

Earth Science

For question 81 on the January 1997 Regents examination in Earth science, there is a discrepancy between data given in the question and data given in the accompanying table. As a consequence, students who selected choice 3 should be given credit as well as students who selected choice 4, the answer indicated on the scoring key.

Sequential Mathematics, Course III

The January 1997 Regents examination in sequential mathematics, course III, has an error in the scoring key for Part II. The scoring key gives an incorrect answer for question 39, *b*(1). The correct answer is $\frac{9}{81}$.

The correct answer key for question 39 appears below.

(39) *a* 12.3 [4]

b (1) $\frac{9}{81}$ [3]

(2) $\frac{9}{81}$ [3]

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, January 28, 1997 — 9:15 a.m. to 12:15 p.m., only

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 3	(11) 7	(21) 2	(31) 4
(2) $2i$	(12) 7	(22) 3	(32) 4
(3) 24	(13) 2	(23) 4	(33) 4
(4) 54	(14) 6	(24) 1	(34) 2
(5) -11	(15) 1	(25) 2	(35) 1
(6) 3	(16) 4	(26) 3	
(7) 216	(17) 1	(27) 4	
(8) III	(18) 2	(28) 2	
(9) 9	(19) 3	(29) 1	
(10) 1	(20) 3	(30) 3	

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* $\pm 3i$ [4]

b $3 \pm i$ [6]

(37) *a* 90, 222, 318 [8]

b 2.1 [2]

(38) *b* 2 [2]

(39) *a* 12.3 [4]

b (1) $\frac{48}{81}$ [3]

(2) $\frac{9}{81}$ [3]

(40) *a* $-x$ [5]

(41) *a* 80 [2]

b 30 [2]

c 12.9 [6]

(42) *a*

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	-2	-1	0	1	2

[4]