HIGH SCHOOL MATHEMATICS: COURSE III—JANUARY 1991 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on the answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1. Simplify and express in terms of i:

$$3\sqrt{-49} - 2\sqrt{-25} - \sqrt{-9}$$

2. A translation maps $A(2, -3)$ onto $A'(-3, -5)$. Under the same translation, find the coordinates of B', the image of $B(1, 4)$.

3. What is the numerical value of the product $(\cos \pi)(\sin \pi)$?

4. Solve for x: $9^{2x-3} = 81$

5. Perform the indicated operation and express in simplest form:

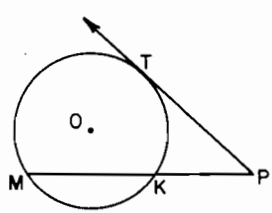
$$\frac{a+3}{a^2-9} \cdot \frac{a^2+a-12}{a+4}$$

6. Solve for x: $6 - \sqrt{x-2} = 2$

7. If point $A\left(\frac{\pi}{2}, 1\right)$ is reflected in the line $y = x$, find the coordinates of the image A'.

8. Evaluate: $\sum_{k=1}^{4} 2k^2$

9. In the accompanying diagram, $\overline{PT}$ is tangent to circle O at T and $\overline{PKM}$ is a secant. If $PK = 4$ and $PM = 9$, find PT.



10. In $\triangle ABC$, $a = 10$, $b = 16$, and $m\angle A = 150$. Find $\sin B$.

11. Express $\frac{3}{2+\sqrt{3}}$ as an equivalent fraction with a rational denominator.

12. If $10^{0.8338} = 6.82$, find the value of $10^{2.8338}$.

13. In a circle with a radius of 2.5 centimeters, a central angle has a measure of 5 radians. What is the length, in centimeters, of the arc intercepted by the central angle?

14. The probability of winning a game is $\frac{1}{3}$ and the probability of losing a game is $\frac{2}{3}$. If the game is played three times, what is the probability of winning exactly twice?

15. Express the product of $(2 + 3i)$ and $(3 + 4i)$ in $a + bi$ form.

Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16. Which combination of letters has both vertical and horizontal line symmetry?
(1) MOM (2) XOOX (3) ZOO (4) NOON

17. For which ordered pair, (x, y), is the fraction $\frac{3}{y - x}$ undefined?
(1) $(-1, 1)$ (2) $(-3, -2)$ (3) $(2, 3)$ (4) $(4, 4)$

18. The solution to $|3x - 4| > 5$ is
(1) $x > 3$ or $x < -\frac{1}{3}$ (3) $x < 3$ and $x > -\frac{1}{3}$
(2) $x \geq 3$ or $x \leq -\frac{1}{3}$ (4) $x \leq 3$ and $x \geq -\frac{1}{3}$

19. What is the area of $\triangle ABC$ if $a = 8$, $b = 6$, and $\sin C = 0.75$?
(1) 9 (2) 18 (3) 36 (4) 72

20. The amplitude of the graph of the equation $y = 4 \sin 2x$ is
(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) 4

21. If placed in standard position, an angle of $\frac{11}{6}\pi$ radians has the same terminal side as an angle of
(1) $-150°$ (2) $-30°$ (3) $150°$ (4) $240°$

22. In a standardized test with a normal distribution of scores, the mean was 75 and the standard deviation was 5.2. Which score could be expected to occur less than 5% of the time?
(1) 95 (2) 85 (3) 75 (4) 65

23. If $\csc \theta = -5$ and $\tan \theta > 0$, then θ must lie in Quadrant
(1) I (2) II (3) III (4) IV

24. Which is the image of A under the transformation $r_{x\text{-axis}} \circ R_{90°}$?
(1) A (2) ∀ (3) ⋗ (4) ≺

25. The expression $\frac{1 - \sin^2 A}{2 \cos A}$ is equivalent to
(1) $\frac{\sin A}{2}$ (2) $\frac{\cos A}{2}$ (3) $\cos \frac{1}{2}A$ (4) $2 \cos A$

26. Cos 280° is equivalent to
(1) $-\sin 80°$ (2) $-\cos 80°$ (3) $\cos 10°$ (4) $\cos 80°$

27. The roots of the equation $x^2 - 4x + 13 = 0$ are
(1) imaginary and unequal (3) real, irrational, and unequal
(2) real, rational, and equal (4) real, rational, and unequal

28. If $\log_x 2 = \frac{1}{3}$, what is the value of x?
(1) $2\frac{1}{3}$ (2) $\sqrt[3]{2}$ (3) 8 (4) 4

29. If $y = \tan(\text{Arc}\cos\frac{1}{2})$, then y equals

(1) 45° (2) 60° (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

30. What is cos 68° 20′?

(1) 0.9315 (2) 0.9293 (3) 0.3638 (4) 0.3692

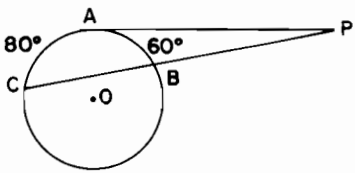
31. What is the inverse of the function $x + 2y + 3 = 0$?

(1) $y = -\frac{1}{2}x - \frac{3}{2}$ (3) $2y + x + 3 = 0$

(2) $y = -2x - 3$ (4) $2x - y + 3 = 0$

32. In the accompanying diagram, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to circle O from point P. If $m\overset{\frown}{AC} = 80$ and $m\overset{\frown}{AB} = 60$, what is the measure of $\angle P$?

(1) 10 (2) 20 (3) 60 (4) 70



33. If $f(x) = x^2 + 2$, what is the value of $f(3i)$?

(1) 11 (2) 8 (3) −7 (4) −4

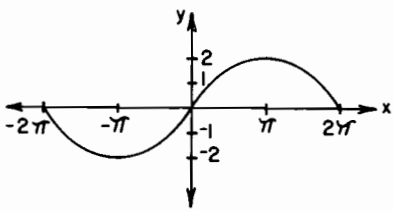
34. Which equation represents the graph?

(1) $y = 2\sin 2x$

(2) $y = \frac{1}{2}\sin x$

(3) $y = 2\sin\frac{1}{2}x$

(4) $y = 2\cos 2x$



35. The third term of the expansion of $(\frac{1}{2}x - 2y)^4$ is

(1) $6x^2y^2$ (2) $-6x^2y^2$ (3) $16xy^3$ (4) $-16xy^3$

Part II

Answer four questions from this part. Show all work unless otherwise directed. [40]

36. *a.* Given: $\log 3 = x$ and $\log 5 = y$.

(1) Express $\log\sqrt{\frac{3}{5}}$ in terms of x and y. [2]

(2) Express $\log 45$ in terms of x and y. [3]

b. Express in simplest form:

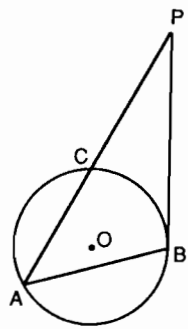
$$\frac{x^2 - 3x}{x^2 + 3x - 10} \cdot \frac{2x + 10}{3} \div \frac{x^2 - x - 6}{x^2 - 4} \quad [5]$$

37. *a.* Given: $\angle A$ and $\angle B$ are obtuse angles, $\sin A = \frac{4}{5}$, and $\cos B = -\frac{5}{13}$. Find, in simplest form, a value for $\tan (A + B)$. [6]
    b. Find all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $\sin 2\theta + \cos \theta = 0$. [4]

38. *a.* Graph and label the function $y = 2^x$ for the restricted domain $-2 \leq x \leq 3$. [3]
    b. On the same set of axes, sketch the reflection of $y = 2^x$ in the x-axis and label the image b. [2]
    c. On the same set of axes, sketch the reflection in the origin of $y = 2^x$ and label it c. [2]
    d. On the same set of axes, sketch the translation of $y = 2^x$ under $T_{(6,2)}$ and label it d. [3]

39. *a.* Write in simplest terms the expansion of $(x + y)^5$. [4]
    b. Evaluate the third term in the expansion of $(x + y)^5$ if $x = \frac{1}{2}$ and $y = \frac{1}{2}$. Express your answer as a fraction in simplest terms. [2]
    c. If a fair coin is flipped five times, what is the probability of getting at least three heads? [4]

40. In the accompanying diagram, tangent $\overline{PB}$ and secant $\overline{PCA}$ are drawn to circle O, $AB = 8\sqrt{2}$, and $m\overset{\frown}{BC} : m\overset{\frown}{AC} : m\overset{\frown}{AB} = 3 : 4 : 5$.



    a. Find:
        (1) $m\overset{\frown}{BC}$ [2]
        (2) $m\angle PAB$ [2]
        (3) $m\angle APB$ [2]
    b. Find the length of $\overline{PB}$. [4]

41. *a.* On the same set of axes, sketch and label the graphs of $y = 2 \sin x$ and $y = \cos 2x$ for the values of x in the interval $-\pi \leq x \leq \pi$. [4, 4]
    b. Based on the graphs drawn in part *a*, which value of x in the interval $-\pi \leq x \leq \pi$ satisfies the equation $2 \sin x - \cos 2x = 3$? [2]

42. *a.* Two consecutive sides of a parallelogram are 6 centimeters and 4 centimeters. If the length of the longer diagonal of the parallelogram is 9 centimeters, find the measure of the largest angle of the parallelogram to the *nearest degree*. [7]
    b. Using your answer to part *a*, find the area of the parallelogram to the *nearest square centimeter*. [3]