HIGH SCHOOL MATHEMATICS: COURSE III—JANUARY 1988 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1. Solve for the *negative* value of w:

$$|7 + 4w| = 13$$

2. Express $140°$ in radian measure.

3. Under a dilation with center P and scale factor k, A' is the image of A. Find k if $PA = 7$ and $PA' = 28$.

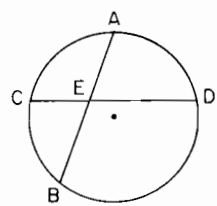
4. If $f(x) = x^2 + 3x$ and $g(x) = x + 3$, for what positive value of x does $f(x) = g(x)$?

5. Evaluate: $\sum_{k=1}^{4} (k^2 + 2)$

6. Evaluate $x^{\frac{3}{4}} - x^0$ if $x = 16$.

7. Express in simplest form: $\dfrac{x - \dfrac{4}{x}}{1 + \dfrac{2}{x}}$

8. In the accompanying diagram, chords $\overline{AB}$ and $\overline{CD}$ intersect in the circle at E. If $m\overset{\frown}{BC} = 60$ and $m\overset{\frown}{AD} = 80$, find $m\angle AEC$.



9. If $\tan \theta = 0.1988$, find θ to the *nearest minute*.

10. A translation maps $(1, 4)$ onto $(7, -3)$. Write the image of $(5, 10)$ under the same translation.

11. What is the value of $\cos\left(\text{Arc sin } \dfrac{\sqrt{3}}{2}\right)$?

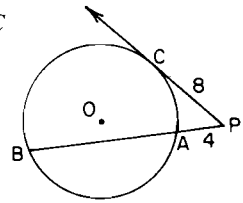
12. Express as a fraction with a rational denominator:

$$\frac{5}{5 - \sqrt{7}}$$

13. Find the value of x if $\log_2 x = 3$.

14. In $\triangle ABC$, $a = \sqrt{2}$, $b = 1$, and $m\angle A = 90$. Find the measure of $\angle B$.

15. In the accompanying diagram, $\overrightarrow{PC}$ is tangent to circle O at C and $\overline{PAB}$ is a secant. If $PC = 8$ and $PA = 4$, find AB.



16. In right triangle ABC, $\overline{AB}$ is the hypotenuse. If $AC = 5$ and $BC = 12$, express $\dfrac{\sin B}{\tan B}$ as a fraction in lowest terms.

17. If $m\angle A = 40$, $a = 6$, and $b = 8$, how many distinct triangles can be constructed?

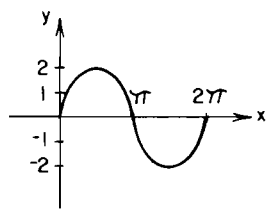
Directions (18–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18. When expressed in scientific notation, the number $0.0000000364 = 3.64 \times 10^n$. The value of n is
(1) 8 (2) 10 (3) -10 (4) -8

19. What is the image of $P(-4, 6)$ under the composite $r_{x=2} \circ r_{y\text{-axis}}$?
(1) $(-8, 6)$ (2) $(4, -2)$ (3) $(6, 0)$ (4) $(0, 6)$

20. The sum of $\sqrt{-8}$ and $2\sqrt{-50}$ is
(1) $12\sqrt{2}$ (2) $-12\sqrt{2}$ (3) $12i\sqrt{2}$ (4) $-12i\sqrt{2}$

21. Which is an equation of the graph shown at the right?
(1) $y = \sin 2x$ (2) $y = 2\cos x$ (3) $y = \cos 2x$
(4) $y = 2\sin x$



22. If the graphs of the equations $y = x^2$ and $y = -2$ are sketched on the same set of axes, the number of points of intersection will be
(1) 1 (2) 2 (3) 3 (4) 0

23. What are the roots of the equation $x^2 - x + 1 = 0$?
(1) $\dfrac{1 \pm \sqrt{3}}{2}$ (2) $\dfrac{1 \pm \sqrt{5}}{2}$ (3) $\dfrac{1 \pm i\sqrt{3}}{2}$ (4) $\dfrac{1 \pm i\sqrt{5}}{2}$

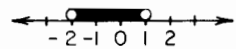
24. Which geometric figure has 72° rotational symmetry?
(1) square (2) regular pentagon (3) rhombus (4) regular hexagon

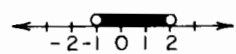
25. If the discriminant of an equation is 10, then the roots are
(1) real, rational, and unequal (2) real, irrational, and unequal (3) real, rational, and equal (4) imaginary

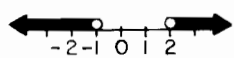
26. The multiplicative inverse of $\sin^2 90° + \cos^2 90°$ equals
(1) 1 (2) 0 (3) -1 (4) ∞

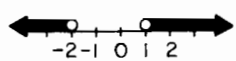
27. In $\triangle ABC$, if $a = 5$, $b = 6$, and $m\angle C = 60$, the value of c is
(1) 1 (2) $\sqrt{31}$ (3) $\sqrt{41}$ (4) $\sqrt{51}$

28. Which is the solution set for the inequality $|2x - 1| < 3$?

(1) 

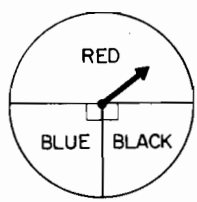
(2) 

(3) 

(4) 

29. What is the value of x in the interval $90° \leq x \leq 180°$ that satisfies the equation $\sin x + \sin^2 x = 0$?
(1) 90° (2) 135° (3) 180° (4) 270°

30. A circular region is divided into three sections and labeled as shown in the accompanying diagram. If the spinner is spun five times, what is the probability that it will land on the red section *exactly* two times?



(1) $\frac{10}{32}$ (2) $\frac{10}{128}$ (3) $\frac{10}{256}$ (4) $\frac{10}{1024}$

31. The value of $\sin(-210°)$ is
(1) $\frac{\sqrt{3}}{2}$ (2) $-\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

32. If $\cos A = \frac{1}{3}$, then the positive value of $\tan \frac{1}{2}A$ is
(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{3}$ (4) $\frac{\sqrt{2}}{2}$

33. The third term in the expansion of $(a - \sqrt{2})^5$ is
(1) $20a^3$ (2) $20a^2\sqrt{2}$ (3) $-20a^2\sqrt{2}$ (4) $40a^3$

34. If $\sin\theta = -\frac{3}{5}$ and $\cos\theta < 0$, then θ terminates in Quadrant
(1) I (2) II (3) III (4) IV

35. What is the domain of the function $f(x) = \sqrt{3x + 7}$?
(1) $\{x | x \geq -\frac{3}{7}\}$ (2) $\{x | x \geq -\frac{7}{3}\}$ (3) $\{x | x \geq \frac{3}{7}\}$ (4) $\{x | x \geq \frac{7}{3}\}$

Part II

Answer four questions from this part. Show all work unless otherwise directed. [40]

36. *a.* On the same set of axes, sketch and label the graphs of the equations $y = \tan x$ and $y = 2 \cos x$ in the interval $0 \leq x \leq 2\pi$. [8]
    b. Using the graphs drawn in part *a*, find the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $\tan x = 2 \cos x$. [2]

37. *a.* Combine and express in simplest form:

$$\frac{y - 20}{y^2 - 16} + \frac{2}{y - 4} \quad [4]$$

    b. Find all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $\sin \theta + 1 = 2 \cos^2 \theta$. [6]

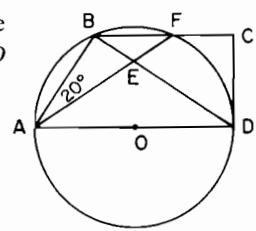
38. *a.* Using logarithms, find $\sqrt[5]{87}$ to the *nearest tenth*. [4]
    b. For all values of θ for which the expressions are defined, prove that the following is an identity:

$$\frac{\tan \theta + \cot \theta}{\cos \theta \sin \theta} = \sec^2 \theta \csc^2 \theta \quad [6]$$

39. In $\triangle ABC$, $a = 30$, $c = 27$, and $m\angle A = 34° 20'$.
    a. Find the measure of $\angle C$ to the *nearest ten minutes*. [6]
    b. Using the answer obtained in part *a*, find the area of $\triangle ABC$ to the *nearest square unit*. [4]

40. *a.* Sketch and label the graph of the equation $y = 2^x$. [2]
    b. Reflect the graph of the equation $y = 2^x$ in the x-axis. Label your answer *b*. [3]
    c. Reflect the graph of the equation $y = 2^x$ in the line $y = x$. Label your answer *c*. [3]
    d. Write an equation of the graph drawn in part *c*. [2]

41. In the accompanying diagram, $\overline{AOD}$ is a diameter of circle O, secant $\overline{CFB}$ is parallel to $\overline{DA}$, $\overline{CD}$ is tangent to circle O at D, chords $\overline{AF}$ and $\overline{BD}$ intersect at E, and $m\angle BAF = 20$. Find:



    a. $m\overset{\frown}{AB}$ [2]
    b. $m\angle BEF$ [2]
    c. $m\angle CDB$ [2]
    d. $m\angle BCD$ [2]
    e. $m\angle CFE$ [2]

42. The table at the right shows the frequency of the average daily temperatures during the month of June.
    a. Using this set of data, find
        (1) the mode [1]
        (2) the median [1]
    b. The mean, $\bar{x}$, for these data is 79. Find the standard deviation to the *nearest tenth*. [8]

Temperature (x_i)	Frequency (f_i)
63	5
70	3
78	4
79	3
80	6
84	4
96	5