The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, June 17, 1993 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

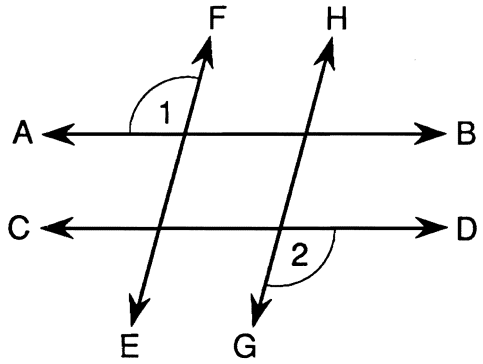
When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$, and $m\angle 1 = 105$. What is $m\angle 2$?



2 The measures of the three sides of a triangle are 6, 8, and 10 centimeters. The midpoints of the three sides are joined to form a second triangle. How many centimeters are in the perimeter of the second triangle?

3 If $\triangle JSO$ is an equilateral triangle, find the measure of an exterior angle at S.

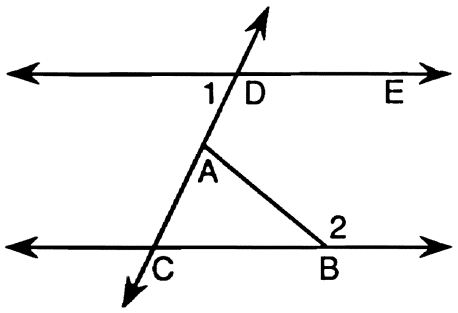
4 If $\square$ is a binary operation defined as $a \square b = \frac{a + b^2}{a}$, evaluate $5 \square 3$.

5 In isosceles triangle ABC, $\overline{AB} \cong \overline{CB}$. If $AB = 2x + 17$, $CB = 4x - 13$, and $AC = 2x + 16$, find the value of x.

6 What is the image of point (4,5) after a reflection in the y-axis?

7 A square has a side of length 3, and a second square has a side of length 4. What is the ratio of the length of a diagonal of the first square to the length of a diagonal of the second square?

8 In the accompanying diagram, $\overleftrightarrow{DE} \parallel \overleftrightarrow{CB}$ and $\overleftrightarrow{CD}$ is a transversal. If $m\angle 1 = 65$ and $m\angle 2 = 140$, find $m\angle CAB$.



9 Solve for x: $\frac{1}{x} + 3 = \frac{7}{2}$

10 Express in radical form the length of the line segment joining the points whose coordinates are (2,4) and (0,−5).

11 In right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$. If $CD = 4$ and $AD = 2$, find DB.

12 The table below defines multiplication on the set $S = \{1, i, -1, -i\}$. Based on the table, what is the value of $i \times i \times i$?

×	1	i	−1	$-i$
1	1	i	−1	$-i$
i	i	−1	$-i$	1
−1	−1	$-i$	1	i
$-i$	$-i$	1	i	−1

13 Write an equation of the line that passes through the origin and is parallel to the line whose equation is $y = 3x - 7$.

14 The coordinates of the midpoint of $\overline{AB}$ are (6,8) and the coordinates of point A are (3,2). Find the coordinates of point B.

Directions (15–34): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 The lengths of two sides of an isosceles triangle are 8 and 10. The length of the third side could be

(1) 6, only
(2) 8, only
(3) 10, only
(4) either 8 or 10

16 What is the image of the point (–3,–1) under the translation that shifts (x,y) to $(x - 2,y + 4)$?

(1) (–1,3)
(2) (–1,–5)
(3) (–5,3)
(4) (–5,–5)

17 What are the factors of $3x^2 + 7x - 20$?

(1) $(3x + 5)(x - 4)$
(2) $(3x - 4)(x + 5)$
(3) $(3x - 5)(x + 4)$
(4) $(3x + 4)(x - 5)$

18 A quadrilateral has diagonals that are congruent but not perpendicular. The quadrilateral contains no right angles. The quadrilateral could be

(1) a square
(2) an isosceles trapezoid
(3) a rectangle
(4) a rhombus

19 Which set contains the number π?

(1) integers
(2) rational numbers
(3) natural numbers
(4) irrational numbers

20 Which is an equation of a line whose slope is equal to zero?

(1) $y = 1$
(2) $x = 2$
(3) $x + y = 5$
(4) $x - y = 3$

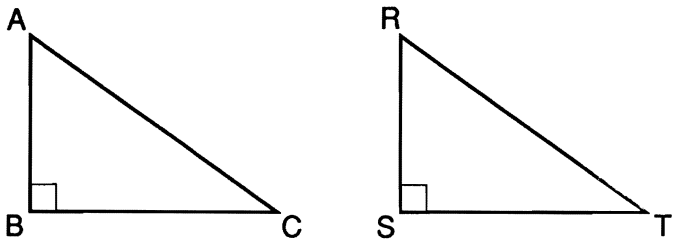
21 The statement $\sim p \vee q$ is equivalent to

(1) $\sim(p \wedge \sim q)$
(2) $\sim(p \vee \sim q)$
(3) $\sim(p \vee q)$
(4) $\sim(p \wedge q)$

22 The vertices of $\triangle ABC$ are $A(0,6)$, $B(3,0)$, and $C(11,0)$. What is the area of $\triangle ABC$ in square units?

(1) 9
(2) 12
(3) 24
(4) 33

23 In the accompanying diagram, $\triangle ABC$ and $\triangle RST$ are right triangles with right angles at B and S, respectively; $\overline{AB} \cong \overline{RS}$ and $\overline{AC} \cong \overline{RT}$.



Which statement can be used to justify that $\triangle ABC \cong \triangle RST$?

(1) SAS ≅ SAS
(2) ASA ≅ ASA
(3) HL ≅ HL
(4) SAA ≅ SAA

24 Which is an equation of a line perpendicular to the line whose equation is $y = -\frac{1}{2}x + 5$?

(1) $y = 2x - 1$
(2) $y = -2x - 1$
(3) $y = \frac{1}{2}x - 1$
(4) $y = -\frac{1}{2}x - 1$

25 If $\cos x = \frac{2}{5}$, what is the measure of $\angle x$, to the *nearest degree*?

(1) 23
(2) 24
(3) 66
(4) 67

26 How many different nine-letter arrangements can be formed from the letters in the word "TENNESSEE"?

(1) $\frac{9!}{3!}$
(2) $\frac{9!}{4!2!2!}$
(3) $\frac{9!}{4 \cdot 2 \cdot 2}$
(4) $\frac{9!}{4!} \cdot \frac{9!}{2!} \cdot \frac{9!}{2!}$

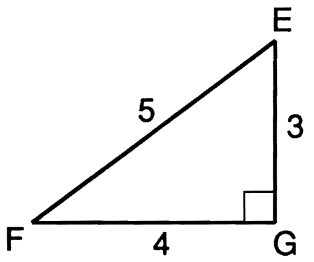
27 An equation of the locus of points 4 units from the point (2,–1) is

(1) $(x - 2)^2 + (y + 1)^2 = 16$
(2) $(x + 2)^2 + (y - 1)^2 = 16$
(3) $(x - 2)^2 + (y - 1)^2 = 16$
(4) $(x + 2)^2 + (y + 1)^2 = 16$

28 What is the y-intercept of the parabola whose equation is $y = x^2 + 5x - 6$?

(1) 1
(2) –1
(3) 6
(4) –6

29 In the accompanying diagram, what is sin E?



(1) $\frac{3}{4}$ (3) $\frac{3}{5}$

(2) $\frac{4}{3}$ (4) $\frac{4}{5}$

30 What are the roots of the equation $x^2 - 4x - 3 = 0$?

(1) 1,3 (3) $2 \pm \sqrt{7}$

(2) $\frac{1}{2}$,3 (4) $-2 \pm \sqrt{7}$

31 Which statement is logically equivalent to the statement, "If we recycle, then the amount of trash in landfills is reduced"?

(1) If we do not recycle, then the amount of trash in landfills is not reduced.
(2) If the amount of trash in landfills is not reduced, then we did not recycle.
(3) If the amount of trash in landfills is reduced. then we recycled.
(4) If we do not recycle, then the amount of trash in landfills is reduced.

32 A circle is inscribed in a square whose sides have length 2. If a dart hits the square, what is the probability that it will hit inside the circle?

(1) 1 (3) $\frac{4}{\pi}$

(2) $\frac{1}{4}$ (4) $\frac{\pi}{4}$

33 How many points do the graphs of the equations $x^2 - y^2 = 16$ and $y = x$ have in common?

(1) 1 (3) 0

(2) 2 (4) 4

34 The expression ${}_{11}C_2$ is equivalent to

(1) ${}_{11}P_2$ (3) ${}_{11}P_9$

(2) ${}_{11}C_9$ (4) $\frac{11!}{2!}$

Directions 35: Leave all construction lines on the answer sheet.

35 *On the answer sheet*, construct a line through C perpendicular to $\overleftrightarrow{ACB}$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]

36 *a* On graph paper, draw the locus of points 3 units from the point (2,3). Label it *a*. [3]

b Write the equation for the locus drawn in part *a*. [2]

c On the same set of axes, draw the image of the graph drawn in part *a* after a reflection in the *x*-axis. Label it *c*. [2]

d On the same set of axes, draw the image of the graph drawn in part *c* after a translation that moves (x,y) to $(x - 2, y + 3)$. Label it *d*. [3]

37 *a* On graph paper, sketch the graph of the function $y = x^2 - 6x + 7$ in the interval $0 \leq x \leq 6$. [5]

b Between which two consecutive integers does the *smaller* root of $x^2 - 6x + 7 = 0$ lie? [2]

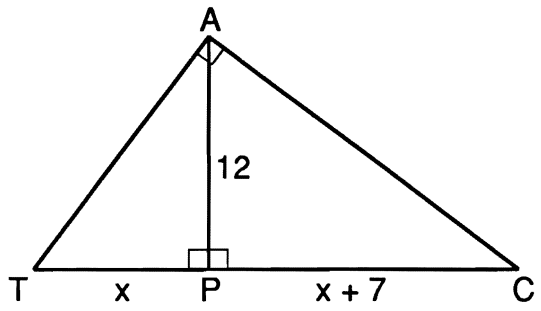
c On the same set of axes, sketch the graph of the equation $y = -2$. [2]

d Determine the number of solutions for the equations $y = x^2 - 6x + 7$ and $y = -2$. [1]

38 Given: $(K \wedge L) \rightarrow M$
$N \rightarrow \sim M$
$L \vee O$
$O \rightarrow P$
K
N

Prove: P [10]

39 In the accompanying diagram of right triangle *CAT*, altitude $\overline{AP}$ divides hypotenuse $\overline{TC}$ into segments of lengths x and $x + 7$, and $AP = 12$.



a Find the length of $\overline{TP}$. [5]

b Find the area of $\triangle CAT$. [2]

c Find the measure of $\angle T$ to the *nearest degree*. [3]

40 A change purse contains nickels, dimes, and quarters. The number of quarters is 8 more than twice the number of dimes, and the number of nickels is 4 less than the number of dimes. The probability of selecting a quarter is $\frac{3}{4}$.

a What is the total value of the coins in the purse? [5]

b Three coins are drawn from the purse.

(1) How many different three-coin selections can be made? [2]

(2) What is the probability that the three coins selected will be one of each kind? [3]

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Quadrilateral $ABCD$ has coordinates $A(0,0)$, $B(6a,3b)$, $C(3a,4b)$, and $D(a,3b)$, $a \neq 0$ and $b \neq 0$.

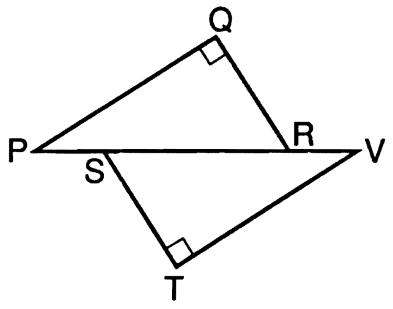
a Using coordinate geometry, show that

(1) $\overline{AB} \parallel \overline{CD}$ [4]

(2) $\overline{AD}$ is *not* parallel to $\overline{BC}$ [4]

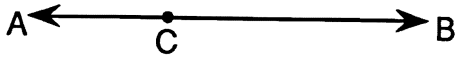
b Which kind of quadrilateral is $ABCD$? Why? [2]

42 Given: $\overline{PSRV}$, $\overline{PS} \cong \overline{VR}$, $\overline{RQ} \parallel \overline{ST}$, $\overline{PQ} \perp \overline{QR}$, and $\overline{VT} \perp \overline{TS}$.



Prove: $\overline{PQ} \cong \overline{VT}$ [10]

35



Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

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Signature

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Tear Here