The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 17, 1992 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

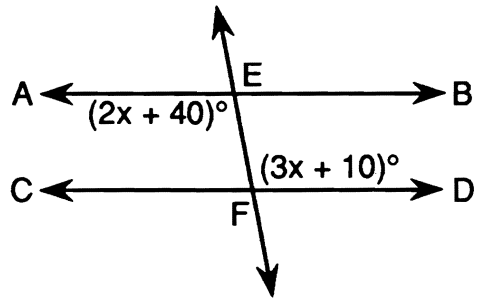
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 A girl 5 feet tall casts a shadow 8 feet long. At the same time, a tree casts a shadow 24 feet long. What is the height, in feet, of the tree?

2 If the binary operation ☺ is defined by $a \text{ ☺ } b = \frac{a + b}{b}$, find the value of $7 \text{ ☺ } 4$.

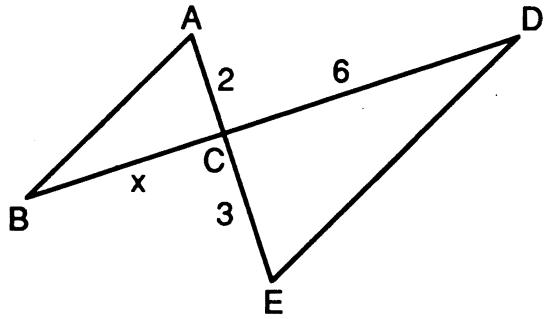
3 In the accompanying diagram, $\overleftrightarrow{AEB} \parallel \overleftrightarrow{CFD}$ and $\overleftrightarrow{EF}$ is a transversal. If $m\angle AEF = 2x + 40$ and $m\angle EFD = 3x + 10$, find x.



4 Point $P(-3,6)$ is reflected in the line whose equation is $y = x$. In which quadrant does the image P' lie?

5 The measure of the vertex angle of an isosceles triangle is three times the measure of a base angle. Find the number of degrees in the measure of the vertex angle.

6 In the accompanying diagram, $\overline{ACE}$, $\overline{BCD}$, and $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$. If $AC = 2$, $CD = 6$, and $CE = 3$, what is the length of $\overline{BC}$?



7 In $\triangle RST$, $m\angle R = 58$ and $m\angle S = 72$. Which is the *shortest* side of the triangle?

8 What is the image of $(-2,5)$ after a reflection in the origin?

9 Find the length, in radical form, of a diagonal of the square whose side is 10.

10 In $\triangle ABC$, $\overline{AC}$ is extended through C to D. If $m\angle BAC = 6x + 10$, $m\angle ABC = 6x - 10$, and $m\angle BCD = 8x + 20$, find x.

11 Given: If the weather is sunny, John goes fishing.
John does not go fishing.
Write a logical conclusion.

12 The midpoint of $\overline{AB}$ is M. If the coordinates of A are $(2,-6)$ and the coordinates of M are $(5,-1)$, find the coordinates of B.

13 Solve for x: $\frac{1}{x} + \frac{1}{6} = \frac{1}{2}$

14 Find the number of square units in the area of the figure formed by the intersection of the lines $y = 4$, $x = 1$, $y = 6$, and $x = 2$.

15 If $\sin 43° = \frac{y}{20}$, what is the value of y to the *nearest tenth*?

16 Find the length of a side of the rhombus whose diagonals measure 12 and 16.

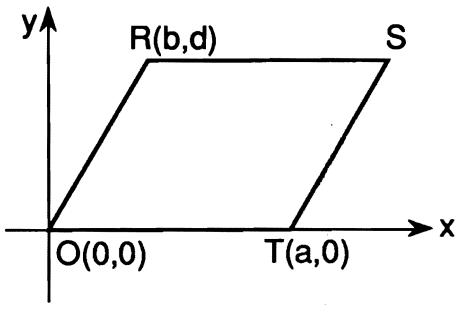
17 Factor completely: $2x^2 - 50$

Directions (18–34): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 Which statement is *always* true?

(1) A quadrilateral is a trapezoid.
(2) A rhombus is a square.
(3) A trapezoid is a parallelogram.
(4) A rectangle is a parallelogram.

19 In the accompanying diagram, three vertices of parallelogram $ORST$ are $O(0,0)$, $R(b,d)$, and $T(a,0)$.



What are the coordinates of S?

(1) (a,b)
(2) (a,d)
(3) $(a+b,d)$
(4) $(a+b,b)$

20 If two cards are drawn from a standard deck of 52 cards without replacement, what is the probability that both cards will be black aces?

(1) $\frac{4}{52} \cdot \frac{4}{51}$
(2) $\frac{2}{52} \cdot \frac{2}{51}$
(3) $\frac{4}{52} \cdot \frac{3}{51}$
(4) $\frac{2}{52} \cdot \frac{1}{51}$

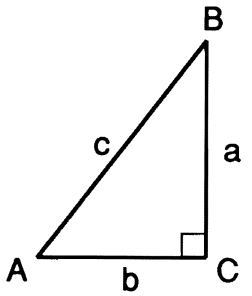
21 An exterior angle at the base of an isosceles triangle is *always*

(1) obtuse
(2) acute
(3) greater than the sum of the base angles
(4) equal to the sum of the base angles

22 Which quadratic equation has 2 and –3 as its roots?

(1) $x^2 - x - 6 = 0$
(2) $x^2 + x - 6 = 0$
(3) $x^2 + 5x + 6 = 0$
(4) $x^2 - 5x + 6 = 0$

23 In the accompanying diagram of right triangle ABC, $\angle C$ is a right angle.



Which equation is valid for $\triangle ABC$?

(1) $\cos A = \frac{c}{b}$
(2) $\tan A = \frac{b}{a}$
(3) $\sin A = \frac{a}{c}$
(4) $\cos B = \frac{a}{b}$

24 Which is an equation of the axis of symmetry for the parabola whose equation is $y = 2x^2 + 8x - 1$?

(1) $x = -2$
(2) $x = 2$
(3) $x = -4$
(4) $x = 4$

25 The expression ${}_9C_6$ is equivalent to

(1) 9!
(2) ${}_9C_3$
(3) ${}_9P_6$
(4) ${}_9P_3$

26 What is the length of the line segment whose endpoints are (1,1) and (3,–3)?

(1) $2\sqrt{5}$
(2) 10
(3) $4\sqrt{2}$
(4) $2\sqrt{2}$

27 Given the true statements: $p \vee q$
p

Which conclusion must follow?

(1) q
(2) $\sim q$
(3) $\sim p$
(4) No valid conclusion is possible.

28 What is the slope of a line perpendicular to the line whose equation is $y = 2x + 7$?

(1) –2
(2) 2
(3) $-\frac{1}{2}$
(4) $\frac{1}{2}$

29 If the probability that an event will occur is $\frac{1}{x}$, then the probability that the event will *not* occur is

(1) x
(2) $\frac{x - 1}{x}$
(3) $\frac{x + 1}{x}$
(4) $\frac{1 - x}{x}$

30 Points M and N lie on line ℓ. Line k is parallel to line ℓ. The total number of points equidistant from points M and N and also equidistant from lines k and ℓ is

(1) 1
(2) 2
(3) 0
(4) 4

31 What are the coordinates of the center of the circle whose equation is $(x - 3)^2 + (y + 7)^2 = 36$?

(1) (−3,−7)
(2) (3,7)
(3) (−3,7)
(4) (3,−7)

32 Given the statement: "A triangle cannot have two right angles." When the indirect method is used to prove this statement, it should be assumed that a triangle

(1) has exactly one right angle
(2) has two right angles
(3) does not have a right angle
(4) does not have two right angles

33 What is the y-intercept of the line whose equation is $3y = 6x + 12$?

(1) 1
(2) 2
(3) 3
(4) 4

34 Using the accompanying table, which is the solution set for $x \otimes x = 6$?

$\otimes$	2	4	6	8
2	2	8	4	6
4	8	8	2	4
6	4	2	6	8
8	4	4	2	6

(1) {2.6}
(2) {2.4}
(3) {6,8}
(4) {2,6,8}

Directions (35): Leave all construction lines on the answer sheet.

35 *On the answer sheet*, construct an angle congruent to $\angle ABC$ using segment DE.

Answers to the following questions are to be written on paper provided by the school.

Part II

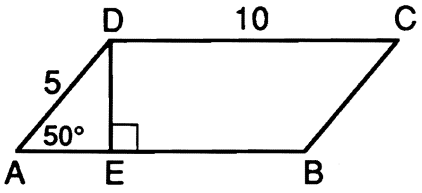
Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]

36 Given: If Lisa plays golf and Lynn plays tennis, then the kids will not go swimming.
Either Lynn plays tennis or she makes dinner for the kids.
If the temperature is over 90°, then the kids will go swimming.
Lisa plays golf.
The temperature is over 90°.

Let G represent: "Lisa plays golf."
Let T represent: "Lynn plays tennis."
Let S represent: "The kids will go swimming."
Let D represent: "Lynn makes dinner for the kids."
Let N represent: "The temperature is over 90°."

Prove: "Lynn makes dinner for the kids." [10]

37 In the accompanying diagram of parallelogram $ABCD$, $m\angle A = 50$, $DC = 10$, and $AD = 5$. Altitude $\overline{DE}$ is drawn.



a Find DE to the *nearest tenth.* [3]

b Find AE to the *nearest tenth.* [3]

c Using the results from parts *a* and *b*, find the area of

(1) $\triangle AED$ to the *nearest integer* [2]

(2) trapezoid $DCBE$ to the *nearest integer* [2]

38 A box contains five wooden rods whose measures are 3, 4, 5, 12, and 13.

a In how many ways can three of these rods be chosen? [2]

b The rods whose measures are 3, 4, and 12 do not form a triangle. List all possible selections of three rods from the set of five rods that do form a triangle. [4]

c Using the answer from part *b*, find the probability that the triangle formed is a

(1) right triangle [2]

(2) scalene triangle [2]

39 Answer both *a* and *b* for all values of x for which these expressions are defined.

a Solve for x and check.

$$\frac{1}{x-1} = \frac{x-1}{4}$$ [5]

b Express the quotient in simplest form:

$$\frac{x^2-4}{x^2+3x-10} \div \frac{x^2+5x+6}{x^2+8x+15}$$ [5]

40 *a* On graph paper, draw the graph of the equation $y = x^2 + 4$, including all values of x in the interval $-3 \leq x \leq 3$. [4]

b Write the coordinates of the turning point of the graph drawn in part *a*. [2]

c Indicate whether the point in part *b* is a minimum or a maximum point. [1]

d On the same set of axes, draw the graph of the image of the graph drawn in part *a* after a reflection in the x-axis. [3]

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Quadrilateral $ABCD$ has coordinates $A(-1,3)$, $B(4,4)$, $C(5,-3)$, and $D(-2,-2)$.

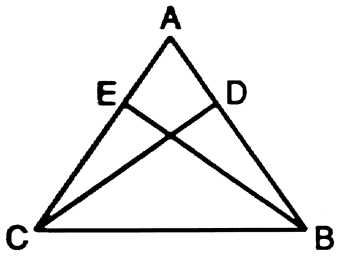
Using coordinate geometry, prove that

a the diagonals are perpendicular [4]

b $ABCD$ has *at least* one pair of congruent sides [3]

c $ABCD$ is *not* a parallelogram [3]

42 Given: $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{CD} \perp \overline{AB}$, and $\overline{BE} \perp \overline{AC}$.



Prove: $\overline{CE} \cong \overline{BD}$ [10]