The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Monday, June 23, 1986 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.   [60]

1 The sides of a triangle are 3, 4, and 5. Find the length of the *shortest* side of a similar triangle whose longest has length 20.
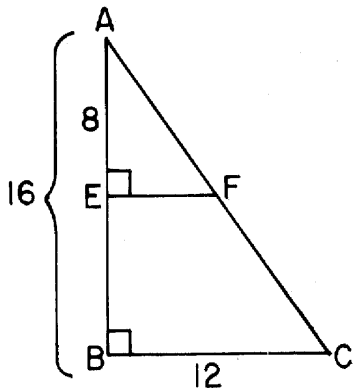
2 In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ is a transversal, and $m\angle 1 = 40$. Find $m\angle 2$.
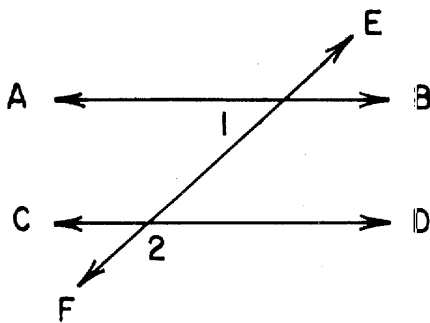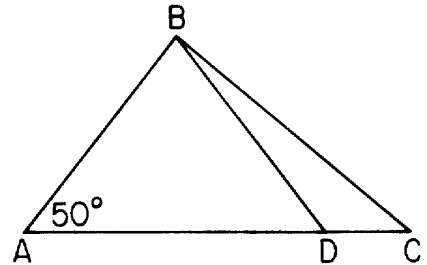

3 Using the accompanying table, find the value of x in the equation $2 \psi x = 4 \psi 3$.

ψ	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

4 In the accompanying diagram of $\triangle ABC$, $\overline{AB} \perp \overline{BC}$ and $\overline{EF} \perp \overline{AB}$ at E. If $BC = 12$, $AB = 16$, and $AE = 8$, find EF.

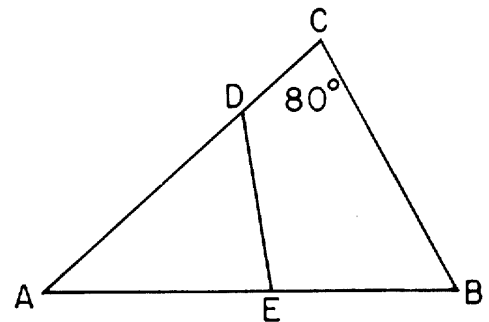5 In the accompanying diagram, $AB = BD$ and $m\angle A = 50$. Find $m\angle BDC$.

6 In parallelogram $ABCD$, $m\angle A = 5x - 20$ and $m\angle C = 3x + 40$. Find the value of x.

7 Using the accompanying table, find the element that does *not* have an inverse.

$\triangle$	a	b	c	d
a	b	a	c	a
b	a	b	c	d
c	c	c	c	a
d	a	d	a	b

8 In the accompanying diagram of $\triangle ABC$, $m\angle B = m\angle ADE$ and $m\angle C = 80$. Find $m\angle AED$.

9 Two line segments have lengths 4 and 11. If the length of a third segment is randomly chosen from the set $\{4,7,8,11\}$, what is the probability the three segments will form a triangle?

10 In parallelogram $ABCD$, the coordinates of A are (4,3) and the coordinates of the midpoint of diagonal $\overline{AC}$ are (2,5). What are the coordinates of C?

11 Evaluate: ${}_{20}C_2$

12 Find the length of a diagonal of the square whose side is 10.

13 Find the positive root of $2x^2 - 5x - 3 = 0$.

14 How many different 5-letter arrangements can be formed from the letters in the word "TEPEE"?

15 Two points whose coordinates are (4,17) and (2,a) determine a line whose slope is 6. Find the value of a.

16 What value of k makes the trinomial $x^2 - 10x + k$ a perfect square?

17 Given rhombus $ABCD$ with coordinates $A(-2,4)$, $B(3,4)$, and $C(6,0)$, find the coordinates of point D.

Directions (18 – 34): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 The table below defines the operation * for the set $S = \{a,b,c,d\}$. What is the value of $(a * b) * (d * c)$?

*	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

(1) a (3) c
(2) b (4) d

19 The coordinates of the vertices of rectangle $ABCD$ are $A(2,2)$, $B(2,6)$, $C(8,6)$, and $D(8,2)$. The area of rectangle $ABCD$ is
(1) 16   (3) 36
(2) 24   (4) 48

20 Which pair of points will determine a line parallel to the x-axis?
(1) (1,3) and (-2,3)   (3) (1,3) and (1,-1)
(2) (1,-1) and (-1,1)   (4) (1,1) and (-3,-3)

21 Segment AB has endpoints $A(-1,3)$ and $B(0,7)$. What is the length of $\overline{AB}$?
(1) $\sqrt{5}$ (3) $\sqrt{17}$
(2) $\sqrt{10}$ (4) $\sqrt{101}$

22 Which is an equation of the line that is parallel to $y = 2x - 8$ and passes through the point $(0,-3)$?

(1) $y = 2x + 3$ (3) $y = -\frac{1}{2}x + 3$

(2) $y = 2x - 3$ (4) $y = -\frac{1}{2}x - 3$

23 Which statement is logically equivalent to $\sim(\sim p \lor q)$?
(1) $p \land q$ (3) $p \lor \sim q$
(2) $\sim p \lor \sim q$ (4) $p \land \sim q$

24 Which quadrilateral must have congruent diagonals?
(1) trapezoid   (3) rhombus
(2) rectangle   (4) parallelogram

25 If two angles of one triangle are congruent to two angles of another triangle, these triangles *must* be
(1) similar   (3) scalene
(2) congruent   (4) isosceles

26 What are the roots of the equation $2x^2 + 3x - 4 = 0$?

(1) $\frac{3 \pm \sqrt{41}}{2}$ (3) $\frac{3 \pm \sqrt{41}}{4}$

(2) $\frac{-3 \pm \sqrt{41}}{2}$ (4) $\frac{-3 \pm \sqrt{41}}{4}$

27 Which is logically equivalent to the statement, "If I live in New York, then I live in the United States"?
(1) If I live in the United States, then I live in New York.
(2) If I do not live in New York, then I do not live in the United States.
(3) If I do not live in the United States, then I do not live in New York.
(4) If I do not live in the United States, then I live in New York.

28 Point P lies between two parallel lines a and b, which are 3 centimeters apart. What is the total number of points equidistant from a and b, and also 2 centimeters from P?
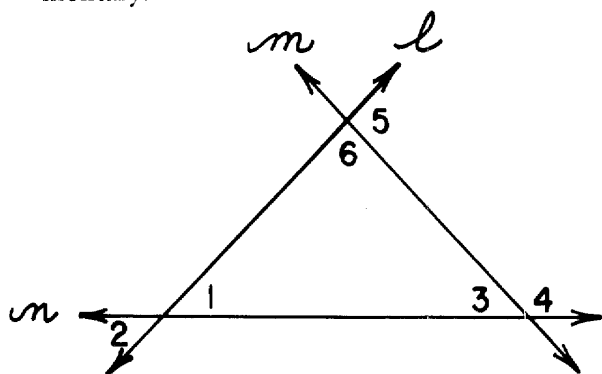(1) 1   (3) 3
(2) 2   (4) 4

29 Which is an equation of the axis of symmetry of the parabola whose equation is
$y = x^2 - 4x + 2$?
(1) $x = -2$ (3) $y = -2$
(2) $x = 2$ (4) $y = 2$

30 In the accompanying figure, ℓ, m, and n are lines with $\ell \perp m$. Which angles are complementary?

(1) 1 and 3   (3) 3 and 4
(2) 1 and 2   (4) 3 and 5

31 An equation of the circle whose center is the origin and which passes through the point (3,0) is
(1) $(x - 3)^2 + y^2 = 9$
(2) $(x - 3)^2 + y^2 = 3$
(3) $x^2 + y^2 = 9$
(4) $x^2 + y^2 = 3$

32 Which set of numbers represents the lengths of the sides of a right triangle?
(1) $\{2,6,\sqrt{40}\}$ (3) $\{4,6,\sqrt{40}\}$
(2) $\{2,18,20\}$ (4) $\{4,36,40\}$

33 Which value of x is a counterexample for the statement $\forall_x \ x(x + 2) > 0$?
(1) 1   (3) -3
(2) 2   (4) -2

34 Which is an equation of the parabola graphed in the accompanying diagram?
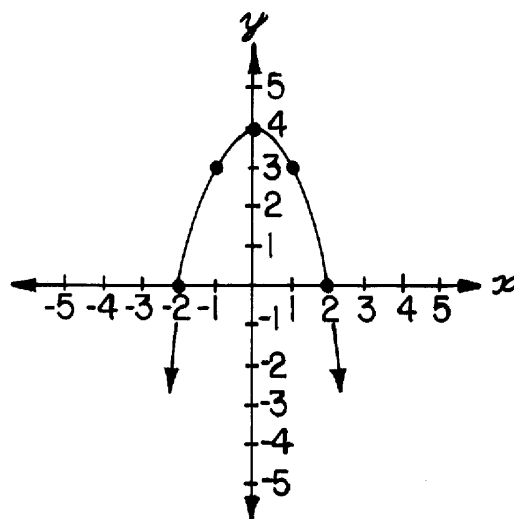
(1) $y = x^2 + 4$ (3) $y = -x^2 + 4$
(2) $y = x^2 - 4$ (4) $y = -x^2 - 4$

Directions (35): Leave all construction lines on the answer sheet.

35 *On the answer sheet,* construct the locus of points equidistant from points A and B.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed. [30]

36 Given the table for operation * as defined on the set $\{D,A,V,E\}$.

*	D	A	V	E
D	V	E	D	A
A	E	D	A	V
V	D	A	V	E
E	A	V	E	D

a What is the identity element? [2]

b What is the inverse of A? [2]

c What is the value of $D * E * D$? [2]

d Solve for x: $x * E = A$ [2]

e Solve for y: $y * y = D$ [1,1]

37 a Describe completely the locus of points n units from the point $P(3,2)$. [3]

b Describe completely the locus of points 2 units from the line whose equation is $x = 3$. [3]

c What is the total number of points that satisfy the conditions in parts a and b simultaneously for the following values of n?

(1) $n < 2$ [2]

(2) $n = 2$ [2]

38 a Find, in radical form, the roots of $x^2 - 6x + 3 = 0$. [4]

b Draw the graph of the equation $y = x^2 - 6x + 3$ including all values of x such that $0 \le x \le 6$. [6]

39 A basket contains 10 pieces of fruit: 5 apples, 3 bananas, and 2 oranges. John selects 3 pieces from this basket for a picnic lunch.

a How many of these 3-piece selections can be made? [2]

b How many of these selections will contain 2 apples and 1 orange? [2]

c How many of these 3-piece selections will contain all apples? [2]

d What is the probability that the 3 pieces selected are all apples? [2]

e What is the probability that the 3 pieces selected are all oranges? [2]

40 In rectangle $ABCD$, the length exceeds the width by 7, and AC exceeds the width by 9. Find the dimensions of the rectangle. [*Only an algebraic solution will be accepted.*] [4,6]

☞ GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41 Given: Marilyn receives good grades in English.
   If Marilyn likes social studies, then she doesn't like computers.
   If Marilyn receives good grades in English and doesn't like geometry, then she likes computers.
   Marilyn likes social studies.

   Let E represent: "Marilyn receives good grades in English."
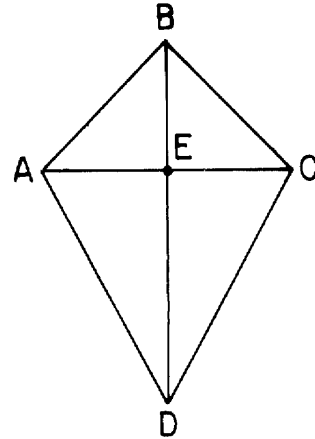
   Let S represent: "Marilyn likes social studies."

   Let C represent: "She likes computers."

   Let G represent: "Marilyn likes geometry."

   Prove: Marilyn likes geometry.   [10]

42 Given: quadrilateral $ABCD$, $\overline{BD}$ intersects $\overline{AC}$ at E, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

   Prove: $\overline{AE} \cong \overline{EC}$ [10]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH — COURSE II

Monday, June 23, 1986 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil. . . . . .Teacher. . . . .

School . . . . .Grade. . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 Answer question 35 on the other side of this sheet.
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

35

•
B

•
A

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____
Signature

FOR TEACHERS ONLY

SCORING KEY
THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Monday, June 23, 1986—1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 18–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 12

(2) 140

(3) 0

(4) 6

(5) 130

(6) 30

(7) c

(8) 80

(9) $\frac{1}{2}$

(10) $(0,7)$ *or* $\begin{array}{l} x = 0 \\ y = 7 \end{array}$

(11) 190

(12) $10\sqrt{2}$ *or* $\sqrt{200}$

(13) 3

(14) 20

(15) 5

(16) 25

(17) $(1,0)$ *or* $\begin{array}{l} x = 1 \\ y = 0 \end{array}$

(18) 2

(19) 2

(20) 1

(21) 3

(22) 2

(23) 4

(24) 2

(25) 1

(26) 4

(27) 3

(28) 2

(29) 2

(30) 1

(31) 3

(32) 1

(33) 4

(34) 3

(35) construction

Part II

Please refer to the Department's pamphlet *Guide for Rating Regents Examinations in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* V   [2]
    b E   [2]
    c E   [2]
    d D   [2]
    e A,E   [1,1]

(37) *a* a circle with center at (3,2)
      and radius n
         or   [3]
      $(x - 3)^2 + (y - 2)^2 = n^2$

    b two lines parallel to and
      two units from $x = 3$
           or   [3]
      $x = 1$ and $x = 5$

    c (1) 0   [2]
       (2) 2   [2]

(38) *a* $\dfrac{6 \pm \sqrt{24}}{2}$ *or* $3 \pm \sqrt{6}$ [4]

(39) *a* 120   [2]
    b 20   [2]
    c 10   [2]
    d $\dfrac{10}{120}$ [2]
    e 0   [2]

(40) Analysis   [4]
      8, 15   [6]