The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, June 16, 1981 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

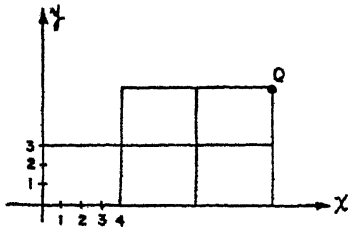
When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 As shown in the accompanying diagram, the dimensions of each congruent rectangle are 4 by 3. What are the coordinates of point Q?



2 The measure of the vertex angle of an isosceles triangle is twice the measure of a base angle. Find the measure of a base angle.

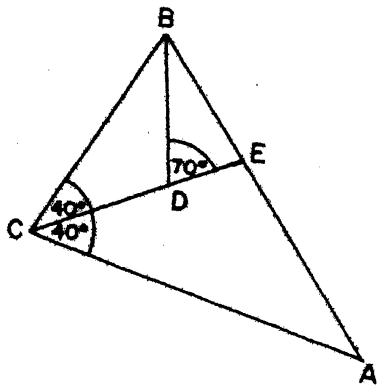
3 Using the accompanying tables, find $3 \@ (4 \# 3)$.

#	1	2	3	4
1	1	2	3	4
2	2	3	4	1
3	3	4	1	2
4	4	1	2	3

@	1	2	3	4
1	1	1	1	1
2	1	2	3	4
3	1	3	4	2
4	1	4	2	3

4 How many different five-letter permutations can be formed from the letters in the word "HAPPY"?

5 In the accompanying diagram of triangle ABC, $\overline{BD}$ bisects $\angle ABC$ and $\overline{CDE}$ bisects $\angle ACB$. If $m\angle BDE = 70$ and $m\angle BCD = 40$, find $m\angle A$.



6 The endpoints of a diameter of a circle are $(-4,-2)$ and $(10,-12)$. Find the coordinates of the center of the circle.

7 Write an equation of the line which passes through the point $(0,4)$ and is perpendicular to the line whose equation is $y = -\frac{1}{2}x + 3$.

8 In $\triangle ABC$, the coordinates of A are $(2,5)$ and the coordinates of B are $(-1,4)$. Find in radical form the length of $\overline{AB}$.

9 Write an equation of the locus of points 5 units from the point $(1,3)$.

10 Evaluate ${}_8C_3$, the number of combinations of 8 items taken 3 at a time.

11 Using the accompanying table, what is the inverse of element P?

#	G	R	O	U	P
G	R	O	P	G	U
R	O	P	U	R	G
O	P	U	G	O	R
U	G	R	O	U	P
P	U	G	R	P	O

12 In right triangle ABC, $\overline{CD}$ is the altitude to the hypotenuse. If $AD = 3$ and $DB = 9$, find AC.

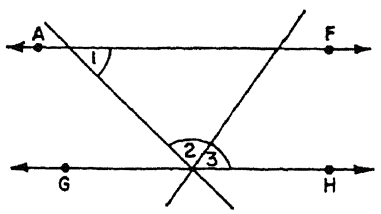
13 A vase contains 6 yellow, 4 red, and 2 pink roses. How many selections of 4 roses will have 2 yellow, 1 red, and 1 pink?

14 The length of a diagonal of a square is $5\sqrt{2}$. What is the length of a side of the square?

15 In $\triangle ABC$, $\overline{AB} \perp \overline{BC}$. What is the probability that $\overline{AB}$ is the longest side of the triangle?

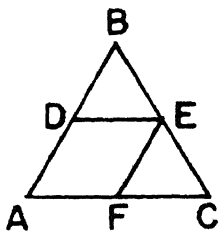
Directions (16–34): For each question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 In the accompanying diagram, $\overleftrightarrow{AF} \parallel \overleftrightarrow{GH}$, $m\angle 1 = 45$, and $m\angle 2 = 80$. What is $m\angle 3$?



(1) 35
(2) 55
(3) 100
(4) 135

17 In the accompanying figure, triangle ABC is an equilateral triangle and $ADEF$ is a rhombus. If D is the midpoint of $\overline{AB}$, and the perimeter of triangle ABC is 12, then what is the perimeter of $ADEF$?



(1) 11
(2) 10
(3) 9
(4) 8

18 Which must have the same truth value as $\sim(p \vee \sim q)$?

(1) $p \wedge \sim q$
(2) $\sim p \wedge q$
(3) $\sim p \vee q$
(4) $\sim p \vee \sim q$

19 What is the slope of a line that passes through the points $A(2,3)$ and $B(-10,8)$?

(1) $-\frac{5}{12}$
(2) $\frac{5}{12}$
(3) $\frac{12}{5}$
(4) $-\frac{12}{5}$

20 The coordinates of the vertices of parallelogram $ABCD$ are $A(0,0)$, $B(5,0)$, $C(8,1)$, and $D(x,1)$. The numerical value of x is

(1) 1
(2) 2
(3) 3
(4) 4

21 The length of one leg of a right triangle is 5 and the length of the hypotenuse is $\sqrt{29}$. What is the length of the other leg?

(1) 24
(2) 2
(3) $\sqrt{24}$
(4) $\sqrt{54}$

22 If the ratio of the corresponding sides of two similar triangles is 4:9, then the ratio of their perimeters is

(1) 16:81
(2) 8:27
(3) 4:9
(4) 2:3

23 If the lengths of two sides of a triangle are 10 and 14, the length of the third side may be

(1) 22
(2) 2
(3) 24
(4) 4

24 Using the set $\{1,3,5,7\}$ and the operation $\odot$ as defined in the accompanying table, which is the solution set of y if $y \odot 5 = 7$?

$\odot$	1	3	5	7
1	3	1	7	5
3	5	3	7	1
5	7	7	1	3
7	1	7	5	3

(1) $\{1\}$
(2) $\{1,3\}$
(3) $\{1,3,5\}$
(4) $\{1,3,5,7\}$

25 If the measure of one angle of a triangle is equal to the sum of the measures of the other two angles, then the triangle is *always*

(1) acute
(2) obtuse
(3) isosceles
(4) right

26 The length of the line segment joining points A and B is 10. There will be only one point equidistant from both A and B and also at a distance q from A if

(1) $q = 10$
(2) $q > 10$
(3) $q = 5$
(4) $5 < q < 10$

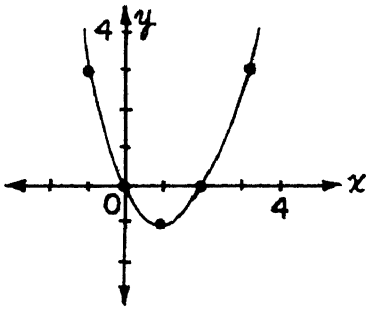
27 The roots of the equation $3x^2 - 6x - 2 = 0$ are

(1) $1 \pm \sqrt{10}$
(2) $\frac{-6 \pm \sqrt{60}}{6}$
(3) $\frac{6 \pm \sqrt{12}}{6}$
(4) $\frac{6 \pm \sqrt{60}}{6}$

28 The statement $[(a \rightarrow \sim b) \wedge (c \rightarrow b)]$ is logically equivalent to

(1) $a \rightarrow c$
(2) $a \rightarrow \sim c$
(3) $\sim a \rightarrow c$
(4) $c \rightarrow a$

29 Which is *not* true for the parabola in the accompanying diagram?



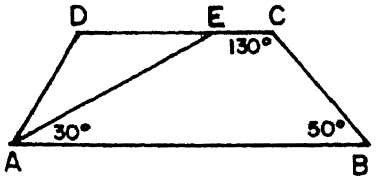
(1) An equation of the axis of symmetry is $x = 1$.
(2) The x-intercepts are at 0 and at 2.
(3) An equation of the parabola is $y = -x^2 + 2x$.
(4) The coordinates of the turning point are $(1,-1)$.

30 Which is the negation of the statement, "All trees have green leaves"?
(1) Some trees have green leaves.
(2) Some trees do not have green leaves.
(3) There is a tree with green leaves.
(4) No tree has green leaves.

31 The difference in degrees between the supplement and the complement of an acute angle is
(1) 30
(2) 60
(3) 90
(4) 150

32 Given the statements, "If we water the lawn in the morning, it will rain in the afternoon," and "It didn't rain in the afternoon." Which statement logically follows?
(1) We didn't water the lawn in the morning.
(2) We watered the lawn in the morning.
(3) It rained in the morning.
(4) It rained in the afternoon.

33 In the accompanying figure, E is a point on $\overline{DC}$ such that $\overline{AE}$ bisects $\angle DAB$. If $m\angle EAB = 30$, $m\angle ABC = 50$, and $m\angle BCD = 130$, what is $m\angle ADC$?



(1) 30
(2) 60
(3) 120
(4) 150

34 The set of integers under the operations of addition and multiplication does *not* form a field because
(1) addition is *not* associative
(2) addition is *not* commutative
(3) multiplication is *not* distributive over addition
(4) some non-zero integers do *not* have inverses under multiplication

Directions (35): Leave all construction lines on the answer sheet.

35 *On the answer sheet*, construct a triangle whose sides are equal in length to a, b, and c.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

36 Given $(Z_4, +, \bullet)$ where $Z_4 = \{0,1,2,3\}$, $+$ is addition clock 4, and $\bullet$ is multiplication clock 4.

a Construct an addition table and a multiplication table for Z_4 as defined. [4]

b What is the additive inverse of 2? [2]

c Find an element which does *not* have a multiplicative inverse. [2]

d Solve for y in the clock 4 system: $(3 \bullet y) + 1 = 3$ [2]

37 The coordinates of the vertices of trapezoid $ABCD$ are $A(3,0)$, $B(7,0)$, $C(7,11)$, and $D(3,8)$. Find:

a the area of the trapezoid [3]

b the perimeter of the trapezoid [3]

c the slope of diagonal $\overline{BD}$ [2]

d the coordinates of the midpoint of diagonal $\overline{AC}$ [2]

38 Solve the following system of equations and check.

$$y = x^2 - x - 6$$
$$y = 2x - 2$$ [8,2]

39 The math department of a certain high school has 5 classes of Course *A*, 4 classes of Course *B*, and 3 classes of Course *C*.

a If a teacher's program consists of 5 classes, how many combinations of 5 classes are possible? [4]

b How many 5-class programs will have 3 Course *A*'s and 2 Course *B*'s? [4]

c If a teacher's program consists of 5 classes, what is the probability that a teacher's program will have 3 Course *A*'s and 2 Course *B*'s? [2]

40 *a* Draw the graph of the equation $y = x^2 + 2x - 8$ for all values of x such that $-5 \leq x \leq 3$. [6]

b What are the coordinates of the turning point of $y = x^2 + 2x - 8$? [2]

c What are the roots of $x^2 + 2x - 8 = 0$? [2]

41 Given: $\triangle ABC$, $\triangle DEF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $AB = 4$, $DF = 6$, $DE = x$ and $AC = x + 5$.

a Write an equation in terms of x which can be used to find DE. [5]

b Find DE. [*Only an algebraic solution will be accepted.*] [5]

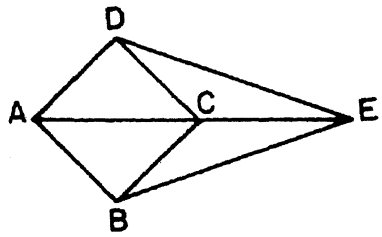
☞ GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed.

42 Given: quadrilateral $ABCD$ with diagonal $\overline{AC}$ extended to point E, $\overline{DE}$ and $\overline{BE}$ are drawn, $\overline{AC}$ bisects $\angle DAB$, $\angle ADC \cong \angle ABC$.



Prove: $\overline{BE} \cong \overline{DE}$ [10]

43 Triangle ABC has vertices $A(-2,1)$, $B(5,2)$, and $C(6,-5)$. Prove that $\triangle ABC$ is an isosceles right triangle. [10]

44 Given: If Peter dates Alice, Ben will be jealous.
If Peter dates Cynthia, Ben will not be jealous.
Either Peter goes to the disco or he dates Alice.
If Peter goes to the disco, he will not meet Elaine.
Peter dates Cynthia.

Let A represent: "Peter dates Alice."
Let B represent: "Ben is jealous."
Let C represent: "Peter dates Cynthia."
Let D represent: "Peter goes to the disco."
Let E represent: "Peter meets Elaine."

a Using A, B, C, D, E, and proper connectives, express *each* statement in symbolic form. [3]

b Prove: "Peter did not meet Elaine." [7]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, June 16, 1981 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) (12,6) *or* $\begin{matrix} x = 12 \\ y = 6 \end{matrix}$

(2) 45

(3) 3

(4) 60

(5) 40

(6) (3,−7) *or* $\begin{matrix} x = 3 \\ y = -7 \end{matrix}$

(7) $y = 2x + 4$

(8) $\sqrt{10}$

(9) $(x - 1)^2 + (y - 3)^2 = 5^2$

(10) 56

(11) *G*

(12) 6

(13) 120

(14) 5

(15) 0

(16) 2

(17) 4

(18) 2

(19) 1

(20) 3

(21) 2

(22) 3

(23) 1

(24) 2

(25) 4

(26) 3

(27) 4

(28) 2

(29) 3

(30) 2

(31) 3

(32) 1

(33) 3

(34) 4

(35) construction

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(36) *a*

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

[4]

b 2 [2]

c 0 *or* 2 *or* both [2]

d 2 [2]

(37) *a* 38 [3]

b 28 [3]

c −2 [2]

d (5,5½) *or* $\begin{array}{l} x = 5 \\ y = 5\frac{1}{2} \end{array}$ [2]

(38) $x = 4,\ y = 6$
$x = -1,\ y = -4$ [8]

(39) *a* 792 [4]

b 60 [4]

c $\frac{60}{792}$ [2]

(40) *b* $(-1,-9)$ *or* $\begin{array}{l} x = -1 \\ y = -9 \end{array}$ [2]

c 2,−4 [2]

(41) *b* 3 [5]

Part III

(44) *a*
$A \rightarrow B$
$C \rightarrow \sim B$
$D \vee A$
$D \rightarrow \sim E$
C [3]