The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 18, 1980 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 In triangle ABC, the measure of angle B is twice the measure of angle A and an exterior angle at vertex C measures $120°$. What is the measure of angle A?

2 If $a \odot b$ is a binary operation defined as $\dfrac{a + b}{a}$, evaluate $2 \odot 4$.

3 The sides of a triangle have lengths of 6, 8, and 10. What is the perimeter of the triangle formed by joining the midpoints of these sides?

4 What is the total number of points that are equidistant from two intersecting lines and are also a distance of 4 centimeters from the point of intersection of the lines?

5 The measures of the three angles of a triangle are in the ratio 1:4:5. What is the number of degrees in the measure of the *smallest* angle?

6 In an isosceles triangle, what is the probability that the altitude and the median drawn to the base are congruent?

7 How many different arrangements of 5 letters can be made using the letters in the word "FLOOR"?

8 The coordinates of the vertices of right triangle ABC are $A(0,4)$, $B(0,0)$, and $C(4,0)$. Find the length of hypotenuse $\overline{AC}$ in radical form.

9 The midpoint M of line segment $\overline{AB}$ has coordinates $(4,9)$. If the coordinates of A are $(2,8)$, what are the coordinates of B?

10 Write an equation of the line whose slope is zero and which passes through the point $(-5,7)$.

11 What is the positive root of the equation $2x^2 + 5x - 3 = 0$?

12 A committee of 5 is to be chosen from 8 club members. How many different committees can be chosen?

13 A signal is made by arranging one red, one white, one blue, and one yellow flag on a vertical pole. What is the probability that the red flag will be on top?

14 What are the coordinates of the center of the circle whose equation is $(x - 3)^2 + (y - 2)^2 = 12$?

15 Using the accompanying table, find the inverse element of b.

| $\square$	a	b	c	d
a	c	d	a	b
b	d	a	b	c
c	a	b	c	d
d	b	c	d	a

16 Given the premises: $p \lor {\sim}q$
$${\sim}p$$
Write a logical conclusion.

17 What value(s) of x will make the following statement true?
$(x^2 = 9) \land (x + 2 = 5)$

Directions (18–34): For each question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 Which is logically equivalent to ${\sim}({\sim}p \land q)$?
(1) $p \land {\sim}q$ (3) ${\sim}p \land {\sim}q$
(2) $p \lor {\sim}q$ (4) ${\sim}p \lor {\sim}q$

19 In the accompanying diagram, $\angle 1$ and $\angle 2$ are supplementary. Which is *always* true?
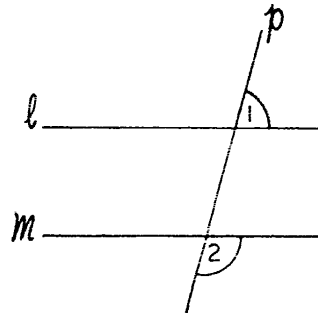
(1) $\ell \perp p$ (3) $\ell \parallel m$
(2) $\ell \perp m$ (4) $p \parallel m$

20 A rectangle has a diagonal of length 10 and one side of length 6. What is the perimeter of the rectangle?
(1) 14   (3) 28
(2) 21   (4) 48

21 In equilateral triangle ABC, $\overline{AD}$ and $\overline{BE}$, the bisectors of angles A and B, respectively, intersect at point F. What is m$\angle AFB$?
(1) 150   (3) 90
(2) 120   (4) 60

22 The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments of lengths 3 and 12. What is the length of this altitude?
(1) 36   (3) 6
(2) 18   (4) 4

23 In isosceles triangle ABC, $\overline{AC} \cong \overline{BC}$ and D is a point lying between A and B on base $\overline{AB}$. If $\overline{CD}$ is drawn, then which is true?
(1) $AC > CD$ (3) m$\angle A >$ m$\angle ADC$
(2) $CD > AC$ (4) m$\angle B >$ m$\angle BDC$

24 Given the true statements, "If Paul catches fish today, then he will give me some." and "Paul will give me some fish."
Which statement must be true?
(1) Paul will not give me some fish.
(2) Paul will not catch some fish today.
(3) Paul will catch some fish today.
(4) No conclusion is possible.

25 Which is an equation of the parabola shown in the accompanying graph?
(1) $y = \frac{1}{2}x^2$
(2) $y = -\frac{1}{2}x^2$
(3) $y = 2x^2$
(4) $y = -2x^2$
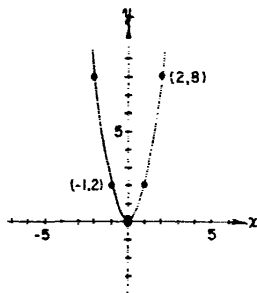
26 An equation which represents the locus of all the points 6 units to the left of the y-axis is
(1) $x = 6$ (3) $y = 6$
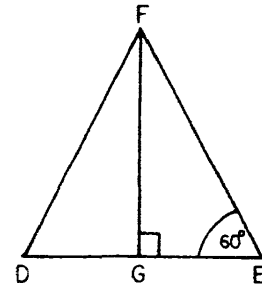(2) $x = -6$ (4) $y = -6$

27 Under which operation is the set $\{1,3,9,27,81,...\}$ closed?
(1) addition   (3) multiplication
(2) subtraction   (4) division

28 Which is not necessary for a system to be a group?
(1) associative property
(2) an identity element
(3) inverse property
(4) commutative property

29 In the accompanying figure, altitude $\overline{FG}$ is drawn in triangle DEF. If $DE = 8$, $DG = 4$, and m$\angle E = 60$, what is the length of $\overline{EF}$?

(1) $\dfrac{8\sqrt{3}}{3}$ (3) 8
(2) $8\sqrt{3}$ (4) $4\sqrt{3}$

30 Which equation has $x = \dfrac{-6 \pm \sqrt{24}}{2}$ as its solution?
(1) $x^2 - 6x - 3 = 0$ (3) $x^2 + 6x - 3 = 0$
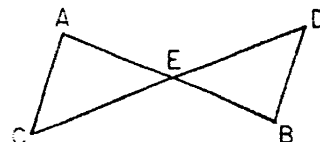(2) $x^2 - 6x + 3 = 0$ (4) $x^2 + 6x + 3 = 0$

31 Which line is parallel to the line $y = 2x + 4$?
(1) $y = 2x + 6$ (3) $y = 4x - 2$
(2) $y = 4 - 2x$ (4) $2y = x - 2$

32 The vertices of a parallelogram are $(0,0)$, $(3,0)$, $(4,4)$, and $(x,4)$. A value of x may be
(1) 1   (3) 3
(2) 2   (4) -1

33 In the accompanying diagram $\overline{AB}$ and $\overline{CD}$ intersect at E and $\angle A \cong \angle B$. Which additional information is needed to show that $\triangle ACE \cong \triangle DBE$?

(1) $\overline{AB} \cong \overline{CD}$ (3) $\overline{AC} \,||\, \overline{BD}$
(2) $\overline{AC} \cong \overline{BD}$ (4) $\angle C \cong \angle D$

34 Which is the negation of the statement. "No grass is brown"?

(1) Some grass is brown.
(2) Some grass is not brown.
(3) All grass is brown.
(4) All grass is not brown.

35 On the answer sheet, construct an equilateral triangle with one vertex at A.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

36 Given the clock 5 (mod 5) field $(F, +, \bullet)$ where $F = \{0,1,2,3,4\}$ and operations $+$ and $\bullet$ are defined below:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\bullet$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

a What is the identity element for $\bullet$? [2]

b Which element does *not* have an inverse under the operation $\bullet$? [2]

c Find the value of $3 + 3 + 3$. [2]

d Find x, if $3x + 4 = 1$. [4]

37 Given: points $A(1,-1)$, $B(5,7)$, $C(0,4)$ and $D(3,k)$.

a Find the slope of $\overleftrightarrow{AB}$. [2]

b Express the slope of $\overleftrightarrow{CD}$ in terms of k. [3]

c If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, find k. [2]

d Write an equation of $\overleftrightarrow{CD}$. [3]

38 In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse $\overline{AB}$, $CD = 12$, and AD exceeds BD by 7.

a If $BD = x$, express AD in terms of x. [1]

b Write an equation in terms of x which can be used to find BD. [3]

c Find BD. [6]

39 a Draw the graph of the equation $y = -x^2 + 2x + 4$, using all integral values of x from $x = -2$ to $x = 4$ inclusive. [6]

b Write an equation of the axis of symmetry. [2]

c Write an equation of the circle whose center is the origin and which passes through the y-intercept of the graph in part a. [2]

40 Ann, Ellen, Fred, Jim, and Mark prepare examinations. A committee of three is to be randomly chosen from them to make up a test.

a How many 3-person committees can be formed? [2]

b What is the probability that Mark will *not* be chosen for the committee? [3]

c How many 3-person committees can be chosen so that Fred and Ann are both members? [3]

d What is the probability that Fred and Ann will both be chosen on the same 3-person committee? [2]

41 On your answer paper, write the letters a through e. Next to each letter, write a true conclusion which can be deduced from each set of statements.

a If it snows this weekend, we will go skiing.
We will not go skiing. [2]

b Either it rains in April or flowers will not grow in May.
It did not rain in April. [2]

c The person who borrowed this book owes the library a quarter.
Mary borrowed this book. [2]

d $\sim r \to s$
$t \to \sim s$ [2]

e $\sim x \to y$
$\sim x$ [2]

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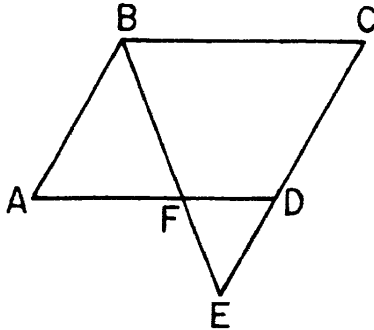
Part III

Answer one question from this part. Show all work unless otherwise directed.

42 Given: $ABCD$ is a parallelogram, $\overline{BFE}$, $\overline{CDE}$, $\overline{AFD}$.

Prove: $AF \times EF = DF \times BF$ [10]

43 Quadrilateral $ABCD$ has vertices $A(2,5)$, $B(7,1)$, $C(2,-3)$, and $D(-3,1)$. Prove by means of coordinate geometry that $ABCD$ is a rhombus.   [10]

44 Given the following statements:

If Carol brings her umbrella, then the weather will be sunny.

If Carol goes to the movies, then the weather is not sunny.

Either Carol goes to the movies or she plays tennis.

Carol brought her umbrella.

Let U represent: "Carol brings her umbrella."
Let S represent: "The weather is sunny."
Let M represent: "Carol goes to the movies."
Let T represent: "Carol plays tennis."

a Using U, S, M, T, and proper connectives, express *each* statement in symbolic form.   [4]

b Using laws of inference, show that Carol played tennis.   [6]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH — COURSE II

Wednesday, June 18, 1980 — 9:15 a.m. to 12:15 p.m., only

ANSWER SHEET

Pupil................ ....Teacher...

School ....Grade......................
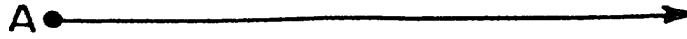
Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1.....	11...................	21...................	31...................
2.	12...................	22...................	32...................
3.....	13................ .	23...................	33...................
4...................	14.............	24...................	34...................
5	15...................	25........	**35 Answer question 35 on the other side of this sheet.**
6...................	16...................	26...................	
7...................	17...........	27...................	
8....	18...................	28...................	
9................. ...	19...................	29...................	
10......	20...................	30...................	

35

A •————————————————————————————→

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination. that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

————————————————————————————
Signature

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 18, 1980 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] For questions 18–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 40	(11) $\frac{1}{2}$	(21) 2	(31) 1
(2) 3	(12) 56	(22) 3	(32) 1
(3) 12	(13) $\frac{1}{4}$	(23) 1	(33) 2
(4) 4	(14) $(3,-2)$ or $\begin{array}{l} x = 3 \\ y = -2 \end{array}$	(24) 4	(34) 1
(5) 18	(15) d	(25) 3	(35) construction
(6) 1	(16) $\sim q$	(26) 2	
(7) 60	(17) 3	(27) 3	
(8) $4\sqrt{2}$ or $\sqrt{32}$	(18) 2	(28) 4	
(9) $(6,10)$ or $\begin{array}{l} x = 6 \\ y = 10 \end{array}$	(19) 3	(29) 3	
(10) $y = 7$	(20) 3	(30) 4	

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(36) *a* 1   [2]
    b 0   [2]
    c 4   [2]
    d 4   [4]

(37) *a* 2   [2]

    b $\dfrac{k-4}{3}$ [3]

    c 10   [2]

    d $y = 2x + 4$ [3]

(38) *a* $x + 7$ [1]

    b $\dfrac{x}{12} = \dfrac{12}{x + 7}$ *or* $x(x + 7) = 144$ [3]

    c 9   [6]

(39) *b* $x = 1$ [2]
    c $x^2 - y^2 = 16$ [2]

(40) *a* 10   [2]

    b $\dfrac{4}{10}$ [3]

    c 3   [3]

    d $\dfrac{3}{10}$ [2]

(41) *a* It will not snow this weekend.   [2]
    b Flowers will not grow in May.   [2]
    c Mary owes the library a quarter.   [2]
    d $t \rightarrow r$ *or* $\sim r \rightarrow \sim t$ [2]
    e y [2]

Part III

(44) *a* $U \rightarrow S$ [1]
      $M \rightarrow \sim S$ [1]
      $M \vee T$ [1]
      U [1]