HIGH SCHOOL MATHEMATICS: COURSE II—JANUARY 1991 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in radical form. [60]

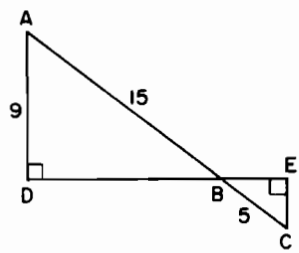
1. Solve the equation $y \mathbin{¢} 5 = 5$ for y in the system defined at the right.

¢	1	3	5	7
1	3	1	7	3
3	1	3	5	7
5	7	5	3	1
7	5	7	1	3

2. What are the coordinates of A', the image of $A(2, 3)$ after a reflection in the x-axis?

3. In parallelogram $ABCD$, $m\angle A = x + 20$ and $m\angle C = 6x - 50$. Find x.

4. In the accompanying figure, $\overleftrightarrow{ABC}$, $\overline{AD} \perp \overline{DBE}$, $\overline{CE} \perp \overline{DBE}$, $AD = 9$, $AB = 15$, and $BC = 5$. Find CE.



5. Given the points $A(2, 3)$, $B(-4, 3)$, $C(5, -1)$, and $D(1, k)$. If $\overline{AB}$ is parallel to $\overline{CD}$, find the value of k.

6. The ratio of the measures of the angles of a triangle is 2:3:5. Find the measure of the *smallest* angle of the triangle.

7. What are the coordinates of the midpoint of the segment whose endpoints are $(-4, 6)$ and $(-8, -2)$?

8. In triangles ABC and DEF, $m\angle C = m\angle F$, $AC = DF$, and $BC = EF$. If $AB = 2x - 1$, $BC = 2x + 5$, and $DE = 5x - 4$, find the value of x.

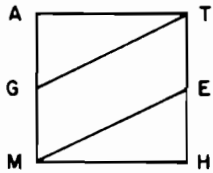
9. In $\triangle ABC$, $m\angle A = 55$ and $\overline{AC} < \overline{BC}$. Which angle is the largest angle of the triangle?

10. Solve for x: $\frac{1}{2} + \frac{5}{x - 2} = 3$

11. In rectangle $ABCD$, $AD = 10$, $CD = 8$, and diagonal $\overline{AC}$ is drawn. Find, to the *nearest degree*, $m\angle CAD$.

12. Evaluate: $\dfrac{{}_4P_2}{{}_4C_2}$

13. In the accompanying diagram, quadrilateral *MATH* is a square, *G* is the midpoint of $\overline{MA}$, *E* is the midpoint of $\overline{TH}$, and $AT = 6$. Find the area of quadrilateral *MGTE*.



14. A translation moves $P(4, 4)$ to $P'(6, 1)$. Find the coordinates of the image of $(-3, 2)$ under the same translation.

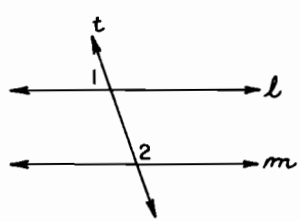
Directions (15–34): For *each* question chosen, write on your answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15. What is the negation of the statement "Quadrilateral *ABCD* is a parallelogram and it has a right angle"?
    (1) Quadrilateral *ABCD* is not a parallelogram or it has a right angle.
    (2) Quadrilateral *ABCD* is a parallelogram and it does not have a right angle.
    (3) Quadrilateral *ABCD* is not a parallelogram or it does not have a right angle.
    (4) Quadrilateral *ABCD* is a parallelogram or it does not have a right angle.

16. Which set of numbers could *not* represent the lengths of the sides of a right triangle?
    (1) {3, 4, 5} (2) {6, 9, 12} (3) {5, 12, 13} (4) {8, 15, 17}

17. Two angles of a triangle have measures of 55° and 65°. Which could *not* be a measure of an exterior angle of the triangle?
    (1) 115° (2) 120° (3) 125° (4) 130°

18. In the accompanying diagram, parallel lines ℓ and m are cut by transversal t. Which statement about angles 1 and 2 *must* be true?
    (1) $\angle 1 \cong \angle 2$.
    (2) $\angle 1$ is a complement to $\angle 2$.
    (3) $\angle 1$ is a supplement to $\angle 2$.
    (4) $\angle 1$ and $\angle 2$ are right angles.



19. If the sum of the measures of two angles of a triangle is equal to the measure of the third angle, the triangle must be
    (1) acute (2) right (3) obtuse (4) scalene

20. What is the distance between points $R(5, 7)$ and $S(-2, 3)$?
    (1) $\sqrt{25}$ (2) $\sqrt{33}$ (3) $\sqrt{55}$ (4) $\sqrt{65}$

21. What is the positive root of the equation $x^2 - 3x - 6 = 0$?
    (1) $\dfrac{3 - \sqrt{33}}{2}$ (2) $\dfrac{3 + \sqrt{33}}{2}$ (3) $\dfrac{-3 - \sqrt{33}}{2}$ (4) $\dfrac{-3 + \sqrt{33}}{2}$

22. How many different six-letter permutations can be formed from the letters of the word "HUBBUB"?
(1) 1 (2) $\frac{6!}{3!2!}$ (3) $\frac{6!}{5!}$ (4) 3!

23. Which is an equation of the locus of points equidistant from the points $A(4, 1)$ and $B(10, 1)$?
(1) $x = 7$ (2) $y = 7$ (3) $x = 3$ (4) $y = 3$

24. If one card is selected at random from a standard deck of 52 cards, what is the probability of choosing a black card or a king?
(1) $\frac{30}{52}$ (2) $\frac{22}{52}$ (3) $\frac{28}{52}$ (4) $\frac{4}{52}$

25. Which is an equation of the line that passes through the point $(-2, 4)$ and is parallel to the line $y = 3$?
(1) $x = -2$ (2) $y = -2$ (3) $x = 4$ (4) $y = 4$

26. If $p \rightarrow q$, $p \vee \sim r$, and r are all true statements, then which statement must also be true?
(1) q (2) $\sim p$ (3) $\sim q$ (4) $\sim r$

27. The slope of a line perpendicular to the line whose equation is $y = 3x - 4$ is
(1) $\frac{1}{3}$ (2) -3 (3) -1 (4) $-\frac{1}{3}$

28. The diagonals of a rhombus are 6 and 4. What is the measure of each side?
(1) $\sqrt{52}$ (2) $\sqrt{13}$ (3) 5 (4) 4

29. What is the sum of the roots of the equation $x^2 - 11x + 10 = 0$?
(1) 11 (2) 7 (3) 10 (4) -7

30. Which is an equation of the locus of points that are 3 units from the point (3, 2)?
(1) $(x - 3)^2 + (y - 2)^2 = 9$ (3) $(y - 2) = (x - 3) + 3$
(2) $(x - 3)^2 + (y - 2)^2 = 3$ (4) $y = 3x - 7$

31. Which is a subset of the set of rectangles?
(1) the set of quadrilaterals (3) the set of squares
(2) the set of rhombuses (4) the set of parallelograms

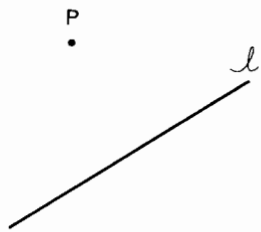
32. Which is an illustration of the associative property?
(1) $x \oplus y = y \oplus x$
(2) $x \oplus (y * z) = (x \oplus y) * (x \oplus z)$
(3) $x \oplus (y \oplus z) = (y \oplus z) \oplus x$
(4) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

33. Which is the turning point of the parabola whose equation is $y = x^2 - 4x + 4$?
(1) $(2, -4)$ (2) $(2, 0)$ (3) $(-2, 16)$ (4) $(-2, 0)$

34. The graphs of the equations $y = x^2$ and $x = 2$ intersect in
(1) 1 point (2) 2 points (3) 3 points (4) 0 points

Directions (35): Leave all construction lines on the answer sheet.

35. *On the answer sheet*, construct a line through point P perpendicular to line ℓ.



Part II

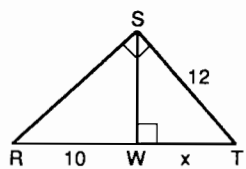
Answer three questions from this part. Show all work unless otherwise directed. [30]

36. *a.* Express in simplest form:

$$\frac{5}{x-1} - \frac{3}{x}, \; x \neq 1, 0 \quad [4]$$

b. The numerator of a certain fraction is three less than the denominator. If the numerator and the denominator are each increased by one, the value of the fraction is $\frac{2}{3}$. Find the original fraction. [*Only an algebraic solution will be accepted.*] [3, 3]

37. In the accompanying diagram of right triangle RST, altitude $\overline{SW}$ is drawn to hypotenuse $\overline{RT}$, $ST = 12$, $RW = 10$, and $WT = x$.



a. Find x. [*Only an algebraic solution will be accepted.*] [7]

b. Find SW to the *nearest integer*. [3]

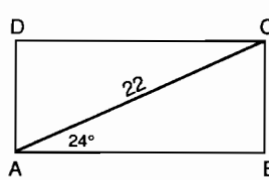
38. Find the area of pentagon $ABCDE$ with vertices $A(-5, 6)$, $B(1, -7)$, $C(6, 0)$, $D(1, 3)$, and $E(1, 6)$. [10]

39. *a.* Sketch the graph of $y = \frac{1}{2}x^2 - 3x + 4$ including all values of x such that $0 \leq x \leq 6$. [6]

b. Find the roots of $\frac{1}{2}x^2 - 3x + 4 = 0$. [2]

c. Find the *smallest* value of $\frac{1}{2}x^2 - 3x + 4$. [2]

40. In the accompanying diagram of rectangle $ABCD$, $AC = 22$ and $m\angle CAB = 24$.



a. Find AB to the *nearest integer*. [4]

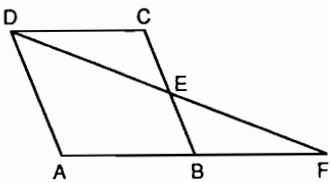
b. Find BC to the *nearest integer*. [4]

c. Using the results from parts *a* and *b*, find the number of square units in the area of $ABCD$. [2]

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41. Given: rhombus $ABCD$, $\overline{DEF}$, $\overline{ABF}$, $\overline{CEB}$, and E is the midpoint of $\overline{DF}$.



Prove: $\overline{AD} \cong \overline{BF}$ [10]

42. Given: Barry is an athlete.
Barry competes in the Empire State Games.
If Barry is an athlete and takes steroids, then he will not win a medal.
If Barry competes in the Empire State Games, then he will win a medal.

Let A represent: "Barry is an athlete."
Let C represent: "Barry competes in the Empire State Games."
Let M represent: "Barry wins a medal."
Let S represent: "Barry takes steroids."

Prove, using the laws of logic, that Barry does not take steroids. [10]

ANSWER KEY—HIGH SCHOOL MATHEMATICS: COURSE II

JANUARY 1991

Part I

1. 3
2. $A'(2, -3)$
3. 14
4. 3
5. -1
6. 36°
7. $(-6, 2)$
8. 1
9. $\angle C$
10. 4
11. 39
12. 2
13. 18
14. $(-1, -1)$
15. 3
16. 2
17. 4
18. 3
19. 2
20. 4
21. 2
22. 2
23. 1
24. 3
25. 4
26. 1
27. 4
28. 2
29. 1
30. 1
31. 3
32. 4
33. 2
34. 1
35. construction

Part II

36. *a.* $\frac{2x + 3}{x^2 - x}$ *or* $\frac{2x + 3}{x(x - 1)}$ [4]

b. $\frac{5}{8}$ [3, 3]

37. *a.* 8 [7]
b. 9 [3]

38. 64 [10]

39. *b.* 2 and 4 [2]
c. $-\frac{1}{2}$ [2]

40. *a.* 20 [4]
b. 9 [4]
c. 180 [2]