The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, January 23, 1990 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form. [60]

1 If $a * b$ is a binary operation defined as $(a + b^2)$, find $4 * 5$.

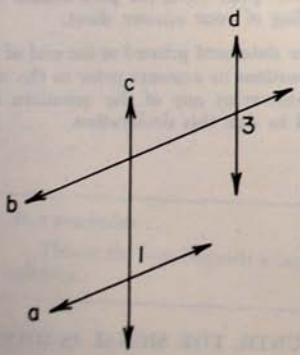
2 Using the accompanying table, solve for x if $x \odot H = C$.

$\odot$	C	A	T	H
C	H	C	A	T
A	C	A	T	H
T	A	T	H	C
H	T	H	C	A

3 Using the accompanying table, find the inverse of the element H.

*	M	A	T	H
M	T	H	M	A
A	H	M	A	T
T	M	A	T	H
H	A	T	H	M

4 In the accompanying figure, $\overleftrightarrow{a} \parallel \overleftrightarrow{b}$ and $\overleftrightarrow{c} \parallel \overleftrightarrow{d}$. If $m\angle 1 = 68$, find $m\angle 3$.



5 In parallelogram $ABCD$, E is the midpoint of $\overline{DC}$ and F is the midpoint of $\overline{AD}$. If $FE = 9$, what is the length of diagonal $\overline{AC}$?

6 How many different five-person committees can be selected from nine people?

7 The measures of the three angles of a triangle are in the ratio 2:3:4. Find the measure of the largest angle of the triangle.

8 Lines ℓ and m are perpendicular. If the slope of line m is $-\frac{4}{3}$, what is the slope of line ℓ?

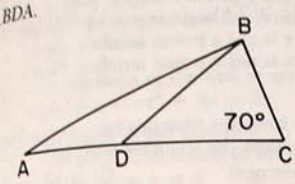
9 The coordinates of the vertices of rhombus $ABCD$ are $A(1,1)$, $B(5,3)$, $C(7,7)$, and $D(3,5)$. Find the coordinates of the point of intersection of the diagonals.

10 In a rectangle, the length is twice the width, and the perimeter is 48. Find the area of the rectangle.

11 In $\triangle ABC$, $m\angle C = 118$ and $m\angle B = 44$. Which is the shortest side of the triangle?

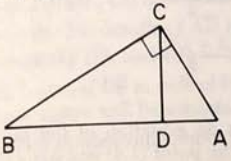
12 In the accompanying diagram of $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{DB}$ and $\overline{DC}$ are angle bisectors, and $m\angle BAC = 20$. Find the measure of $\angle BDC$.

13 In the accompanying diagram of $\triangle ABC$, $\overline{BD}$ is drawn so that $\overline{BD} \cong \overline{DC}$. If $m\angle C = 70$, find $m\angle BDA$.



14 A bag contains five green marbles and three red marbles. If three marbles are chosen at random without replacement, what is the probability that all three will be green?

15 In the accompanying diagram of $\triangle ABC$, $m\angle ACB = 90$ and $\overline{CD}$ is an altitude. If $AD = 2$ and $DB = 6$, find AC.



16 The diagonals of a rhombus have lengths of 12 centimeters and 16 centimeters. Find the number of centimeters in the length of one side of the rhombus.

17 The graphs of the equations $x^2 + y^2 = 9$ and $x = 1$ are drawn on the same set of axes. What is the total number of points common to both graphs?

Directions (18–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 If a card from a standard deck of 52 cards is drawn, the probability of choosing a face card or an ace is

(1) $\frac{16}{52}$
(2) $\frac{12}{52}$
(3) $\frac{8}{52}$
(4) $\frac{4}{52}$

19 How many points are equidistant from two intersecting lines and 3 units from their point of intersection?

(1) 1
(2) 2
(3) 3
(4) 4

20 Which statement is logically equivalent to $\sim(\sim p \vee q)$?

(1) $p \vee \sim q$
(2) $p \wedge \sim q$
(3) $\sim p \vee \sim q$
(4) $\sim p \wedge \sim q$

21 The coordinates of the turning point of the graph of $y = x^2 + 4x + q$ are $(-2,-7)$. The value of q is

(1) -1
(2) -2
(3) -3
(4) -17

22 What is the distance between the points $(-1,2)$ and $(2,6)$?

(1) 5
(2) 25
(3) $\sqrt{17}$
(4) $\sqrt{73}$

23 Which statement is *not* true for all parallelograms?

(1) Opposite sides are parallel.
(2) Opposite sides are congruent.
(3) The diagonals bisect each other.
(4) The diagonals are congruent.

24 How many different five-letter permutations can be formed from the letters of the word "DITTO"?

(1) 5!
(2) $(5 - 2)!$
(3) $\frac{5!}{2!}$
(4) ${}_5P_2$

25 What is the negation of the statement "Some parallelograms are squares"?

(1) All parallelograms are not squares.
(2) Some squares are parallelograms.
(3) Some parallelograms are not squares.
(4) All squares are parallelograms.

[OVER]

26 Given the equation $x^2 - 8x + 15 = 0$. Which statement is true?
(1) The sum of the roots is 15.
(2) Both roots are greater than zero.
(3) One root is less than zero and the other root is greater than zero.
(4) One root is zero and the other root is greater than zero.

27 An equation of the circle whose center is (–3,1) and whose radius is 8 is
(1) $(x - 3)^2 + (y + 1)^2 = 64$
(2) $(x - 3)^2 + (y + 1)^2 = 8$
(3) $(x + 3)^2 + (y - 1)^2 = 64$
(4) $(x + 3)^2 + (y - 1)^2 = 8$

28 Which expression is *not* equivalent to ${}_8C_5$?
(1) 56
(2) ${}_8P_5$
(3) ${}_8C_3$
(4) $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$

29 Which set of numbers could represent the lengths of the sides of a triangle?
(1) {9,16,20}
(2) {8,11,19}
(3) {3,4,8}
(4) {11,5,5}

30 Which is an equation of the line that passes through the point (–1,5) and is parallel to the y-axis?
(1) $y = -1$
(2) $y = 5$
(3) $x = -1$
(4) $x = 5$

31 Which statement is logically equivalent to the statement "If $x = 3$, then x is a prime number"?
(1) If x is a prime number, then $x = 3$.
(2) If $x \neq 3$, then x is not a prime number.
(3) If x is not a prime number, then $x \neq 3$.
(4) If x is not a prime number, then $x = 3$.

32 If two isosceles triangles have congruent vertex angles, then the triangles must be
(1) congruent
(2) right
(3) equilateral
(4) similar

33 Which set is *not* closed under addition?
(1) natural numbers
(2) even integers
(3) whole numbers
(4) odd integers

34 What are the roots of the equation $x^2 - 5x - 2 = 0$?
(1) $x = \frac{5 \pm \sqrt{17}}{2}$
(2) $x = \frac{5 \pm \sqrt{33}}{2}$
(3) $x = \frac{-5 \pm \sqrt{17}}{2}$
(4) $x = \frac{-5 \pm \sqrt{33}}{2}$

35 Which is an equation of the line that passes through the point (1,4) and has a slope of 3?
(1) $y = 3x + 4$
(2) $y = \frac{1}{3}x + 4$
(3) $y = 3x - 1$
(4) $y = 3x + 1$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed. [30]

36 *a* Write an equation of the axis of symmetry of the graph of $y = -x^2 + 8x - 7$. [2]

b Draw the graph of the equation $y = -x^2 + 8x - 7$, including all integral values of x such that $0 \leq x \leq 8$. [6]

c From the graph drawn in part *b*, find the roots of $-x^2 + 8x - 7 = 0$. [2]

37 Solve the following system of equations algebraically and check:

$$y = 2x^2 + 2x + 3$$
$$x = y - 3$$

[8,2]

38 In right triangle ABC, $\overline{CD}$ is the altitude drawn to hypotenuse $\overline{AB}$. The length of $\overline{AD}$ is 2 units less than the length of $\overline{DB}$, and $CD = 3$.

a Find the length of $\overline{DB}$ in radical form. [*Only an algebraic solution will be accepted.*] [4,4]

b In this triangle, which statement is true? [2]

(1) $CD < DB$
(2) $CD = DB$
(3) $CD > DB$

39 The coordinates of the vertices of $\triangle XYZ$ are $X(1,1)$, $Y(12,-1)$, and $Z(9,5)$.

a Prove that $\triangle XYZ$ is a right triangle. [5]

b Find the area of $\triangle XYZ$. [5]

40 In a given plane, P is a point on line ℓ.

a Describe fully the locus of points in the plane 3 units from line ℓ. [3]

b Describe fully the locus of points in the plane h units from point P. [3]

c Using the loci described in parts *a* and *b*, what is the number of points of intersection for the following values of h?

(1) $h = 1$ [1]
(2) $h = 3$ [1]
(3) $h = 3.6$ [2]

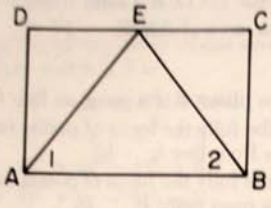
☞ GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41 Given: rectangle $ABCD$ with E, the midpoint of $\overline{DC}$.



Prove: $\angle 1 \cong \angle 2$ [10]

42 Given: Either the Lakers won the game or the Pistons won the game.

If Isiah was in the game and Magic was in the game, then Kareem was *not* in the game.

If the Pistons won the game, then Isiah was in the game.

Kareem was in the game.

Magic was in the game.

Let L represent: "The Lakers won the game."
Let P represent: "The Pistons won the game."
Let I represent: "Isiah was in the game."
Let M represent: "Magic was in the game."
Let K represent: "Kareem was in the game."

Prove: The Lakers won the game. [10]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, January 23, 1990 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 18–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 29

(2) T

(3) A

(4) 112

(5) 18

(6) 126

(7) 80

(8) $\frac{3}{4}$

(9) (4,4)

(10) 128

(11) $\overline{BC}$ *or* a

(12) 100

(13) 140

(14) $\frac{5}{28}$

(15) 4

(16) 10

(17) 2

(18) 1

(19) 4

(20) 2

(21) 3

(22) 1

(23) 4

(24) 3

(25) 1

(26) 2

(27) 3

(28) 2

(29) 1

(30) 3

(31) 3

(32) 4

(33) 4

(34) 2

(35) 4

[OVER]

Part II

Please refer to the Department publication *Guide for Rating Regents Examinations in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* $x = 4$ [2]

c 1,7 [2]

(37) (0,3) and $(-\frac{1}{2}, \frac{5}{2})$ [8]

(38) *a* $1 + \sqrt{10}$ [4,4]

b 1 [2]

(39) *b* 30 [5]

(40) *a* two lines parallel to ℓ and 3 units on either side [3]

b a circle with center P and radius h [3]

c (1) 0 [1]

(2) 2 [1]

(3) 4 [2]

Notice . . .

The January 1990 Regents examination in Course II, Three-Year Sequence for High School Mathematics, will be the last examination based on the original 1977 Syllabus. The June 1990 examination will be based on the revised syllabus (1989). If you have not received a copy of the revised syllabus, contact Fredric Paul, Chief, Bureau of Mathematics Education, State Education Department, Albany, NY 12234.