HIGH SCHOOL MATHEMATICS: COURSE I—JUNE 1991 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1. The mean of a set of 5 numbers is 10. If all the numbers are doubled, what is the mean of this new set of numbers?

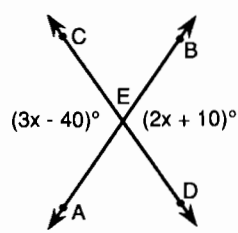
2. Solve for x: $0.3x + 1.7 = 2$

3. Find the value of $a^2 - b$ if $a = 3$ and $b = -4$.

4. Solve for x terms of a and b:

$$2x + a = b$$

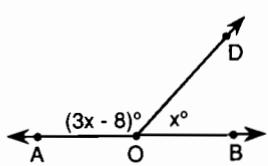
5. In the accompanying diagram, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E. If $m\angle AEC = 3x - 40$ and $m\angle BED = 2x + 10$, find the value of x.



6. If the measures of the angles of a triangle are in the ratio 1:2:3, find the number of degrees in the *smallest* angle.

7. If x varies directly as y and $x = 60$ when $y = 5$, find the value of y when $x = 36$.

8. In the accompanying diagram, $\overleftrightarrow{AOB}$ is a straight line, $m\angle AOD = 3x - 8$, and $m\angle BOD = x$. Find x.



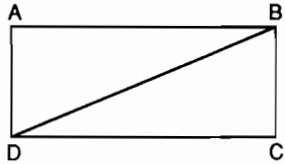
9. Write, in symbolic form, the inverse of $p \rightarrow \sim q$.

10. Find the area of the triangle whose vertices have coordinates (8, 0), (0, 10), and (0, 0).

11. Solve for x: $\frac{2}{3}x - 2 = 10$

12. Express $\frac{5a}{6} - \frac{4a}{9}$ as a single fraction in simplest form.

13. In the accompanying diagram, *ABCD* is a rectangle. If $DB = 26$ and $DC = 24$, find BC.



14. Solve for the positive value of x:

$$x^2 - 5x - 24 = 0$$

15. Express $2x^2 - x - 3$ as the product of two binomials.

16. From $5x^2 + 3x - 6$ subtract $4x^2 - 5x + 6$.

17. Evaluate: ${}_7P_3$

18. The cumulative frequency table shows the distribution of scores on a math test. How many scores were greater than 90?

Interval	Cumulative Frequency
61–70	4
61–80	10
61–90	12
61–100	16

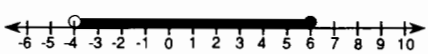
Directions (19–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

19. A rotation of a figure can be considered
    (1) a turning of the figure about some fixed point
    (2) a slide of the figure
    (3) an enlargement or a reduction of the figure
    (4) a mirror image of the figure

20. A tree 24 feet tall casts a shadow 16 feet long at the same time a man 6 feet tall casts a shadow x feet long. What is the length of the man's shadow?
    (1) 6 (2) 5 (3) 3 (4) 4

21. If a letter is chosen at random from the word "BASEBALL," what is the probability that the letter chosen is *not* an "L"?
    (1) $\frac{1}{8}$ (2) $\frac{2}{8}$ (3) $\frac{6}{8}$ (4) $\frac{7}{8}$

22. Which inequality is represented by the accompanying graph?



    (1) $-4 \leq x \leq 6$ (3) $-4 \leq x < 6$
    (2) $-4 < x < 6$ (4) $-4 < x \leq 6$

23. The quotient of $\frac{14x^6y}{2x^2y}$, $x \neq 0$, $y \neq 0$, is
    (1) $7x^3$ (2) $7x^4$ (3) $7x^3y$ (4) $7x^4y$

24. The expression $\frac{5}{2x - 10}$ is undefined when x is equal to
(1) 0 (2) -5 (3) 5 (4) 10

25. A quadrilateral with exactly one pair of parallel sides is a
(1) rhombus (2) rectangle (3) square (4) trapezoid

26. When drawn on the same set of axes, the graph of the equations $y = x + 1$ and $y + x = 3$ intersect at the point whose coordinates are
(1) (2, 1) (2) (1, 2) (3) (2, 3) (4) $(-1, 4)$

27. If the radius of a circle is doubled, then the circumference of the circle is multiplied by
(1) $\frac{1}{2}$ (2) 2 (3) 16 (4) 4

28. The number of feet in c inches is
(1) $\frac{c}{12}$ (2) $\frac{c}{36}$ (3) $\frac{12}{c}$ (4) $12c$

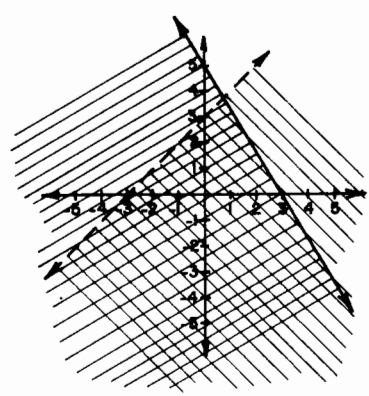
29. Triangle $A'B'C'$ is the image of $\triangle ABC$ under a dilation such that $A'B' = \frac{1}{2}AB$. Triangles ABC and $A'B'C'$ are
(1) congruent but not similar
(2) similar but not congruent
(3) both congruent and similar
(4) neither congruent nor similar

30. The perimeter of a square is $4a$. What is the area of the square?
(1) a^2 (2) $4a^2$ (3) 16 (4) 4

31. Let p represent "$x > 10$" and let q represent "x is a multiple of 5." Which is true if $x = 26$?
(1) $p \vee q$ (2) $p \rightarrow q$ (3) $p \wedge q$ (4) $p \leftrightarrow q$

32. The sum of $\sqrt{50}$ and $\sqrt{2}$ is
(1) $\sqrt{52}$ (2) 10 (3) $6\sqrt{2}$ (4) 12

33. Which ordered pair is in the solution set of the system of inequalities shown in the accompanying graph?
(1) (0, 0) (2) (1, 5) (3) $(-3, 3)$ (4) (3, 3)



34. Which statement would be a correct heading for column 3 in the accompanying table?
(1) $p \leftrightarrow q$
(2) $\sim(p \vee q)$
(3) $\sim p \rightarrow q$
(4) $\sim(p \wedge q)$

Column 1	Column 2	Column 3
p	q	?
T	T	F
T	F	T
F	T	T
F	F	T

35. If the ratio of the edges of two cubes is 2:3, the ratio of the two volumes is
(1) 2:3 (2) 4:9 (3) 8:27 (4) 2:5

Part II

Answer four questions from this part. All work, including calculations, must be shown on your answer paper. [40]

36. *a.* On the same set of coordinate axes, graph the following system of inequalities:

$$\begin{aligned} y &\geq -3 \\ 2y - x &< 6 \end{aligned} \quad [8]$$

b. Write the coordinates of a point in the solution set of the system of inequalities graphed in part *a*. [2]

37. If 3 is added to twice the square of an integer, the result is equal to seven times the integer. Find the integer. [*Only an algebraic solution will be accepted.*] [4, 6]

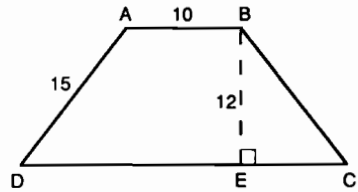
38. The table shows the results of a math test given to a number of students.

Interval	Frequency
96–100	9
91–95	7
86–90	9
81–85	8
76–80	6
71–75	5

a. Draw a frequency histogram based on the data. [4]
b. In which interval is the median score? [2]
c. How many students scored at or below the 25th percentile? [2]
d. To get an A on this test, a student had to have a score greater than 90. What is the probability that a student selected at random from this distribution got an A on the test? [2]

39. One black marble and two red marbles are in a bag. Erika picks a marble from the bag at random. She looks at it, returns it, and makes a second random selection.
a. Draw a tree diagram or list the sample space showing all possible outcomes. [2]
b. What is the probability that two red marbles were selected? [2]
c. What is the probability that two black marbles were selected? [2]
d. What is the probability that one black and one red marble were selected? [2]
e. What is the probability that *at most* one black marble was selected? [2]

40. In the accompanying diagram, $ABCD$ is an isosceles trapezoid with altitude $\overline{BE}$, $AB = 10$, $AD = 15$, and $BE = 12$.



a. Find EC. [2]

b. Find the area of
(1) triangle BEC [2]
(2) trapezoid $ABCD$ [3]
(3) trapezoid $ABED$ [1]

c. If diagonal $\overline{DB}$ is drawn, find the area of $\triangle ABD$. [2]

41. Use any method [algebraic, trial and error, making a table, etc.] to solve this problem. A written explanation of how you arrived at your answer is also acceptable. Show all work.

There are two pairs of integers that satisfy both of these conditions:

The larger integer is 9 more than the smaller integer.
The sum of the squares of the integers is 41.

a. Find the two pairs of integers. [8]

b. Show that one pair of integers found in part *a* satisfies both given conditions. [2]

42. Solve the following system of equations algebraically and check:

$$\begin{aligned} 4x + 3y &= 25 \\ 5x + 2y &= 33 \end{aligned} \quad [8, 2]$$