HIGH SCHOOL MATHEMATICS: COURSE I—JUNE 1984 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

1. Solve for m: $\frac{m}{4} = \frac{5}{2}$

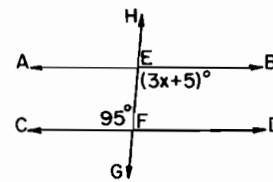
2. Solve for x: $5 + 3(x + 2) = 14$

3. Find the value of $3x^2y$ if $x = -2$ and $y = 3$.

4. Jean is making sandwiches for a class picnic. She is using 4 different fillings with 2 different kinds of bread. How many different kinds of sandwiches can she make using one kind of filling on one kind of bread for each sandwich?

5. If all seven of the letters of the word "REGENTS" were placed in a hat, what would be the probability of drawing an E at random on the first draw?

6. In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{HG}$ intersects $\overleftrightarrow{AB}$ at E and $\overleftrightarrow{CD}$ at F. If $m\angle CFE = 95$ and $m\angle BEF = 3x + 5$, find x.



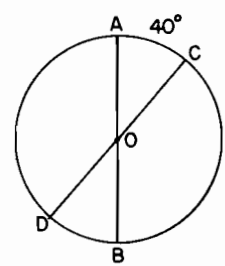
7. A student received test scores of 89, 86, 81, 94, and 75. Find the mean score.

8. If a card is drawn at random from a standard deck of 52 cards, what is the probability that the card is a red queen?

9. The measure of each base angle of an isosceles triangle is 15°. Find the number of degrees in the measure of the vertex angle.

10. Solve the following system of equations for x: $4x - y = 10$
$x + y = 5$

11. In the accompanying diagram of circle O, diameters AB and CD intersect at O. If the measure of arc AC is 40°, find the number of degrees in the measure of angle COB.

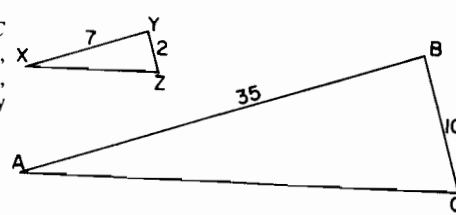


12. Factor: $x^2 + 5x - 6$

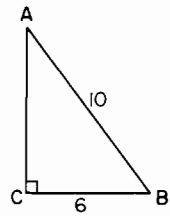
13. Solve for x: $0.4x - 1.7 = 0.7$

14. Solve for x: $\frac{x}{3} - 2 = 10$

15. In the accompanying diagram, triangle ABC is similar to triangle XYZ, with $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$. If $AB = 35$, $XY = 7$, $BC = 10$, and $YZ = 2$, how many times larger is side AC than side XZ?



16. In the accompanying diagram, $\triangle ABC$ is a right triangle with the right angle at C. If $AB = 10$ and $BC = 6$, find AC.



17. Write, in symbolic form, using p and q, the converse of $\sim p \rightarrow \sim q$.

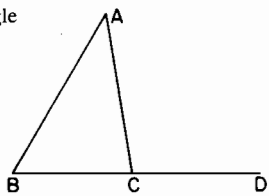
18. What is the slope of the graph of the equation $y = -x + 1$?

19. From $3x^2 + x - 2$ subtract $x^2 - 2x + 3$.

20. Find the positive root of the equation $x^2 - 25 = 0$.

21. Two complementary angles are in the ratio of 8:1. Find the number of degrees in the measure of the smaller angle.

22. In the accompanying diagram, $\angle ACD$ is an exterior angle of triangle ABC. If $m\angle A = 40$ and $m\angle B = 60$, find $m\angle ACD$.



Directions (23–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

23. The statement, "x is odd and x is greater than 2," is true when x equals
(1) 1 (2) 2 (3) 3 (4) 4

24. Let s represent "You score at least 65," and let p represent "You pass the exam." Which is the symbolic representation of the statement, "If you do not score at least 65, then you do not pass the exam"?
(1) $\sim s \wedge \sim p$ (2) $\sim s \rightarrow \sim p$ (3) $s \rightarrow p$ (4) $\sim p \rightarrow \sim s$

25. If n represents an odd integer, which represents the next larger consecutive odd integer?
(1) $n - 1$ (2) $2n$ (3) $n + 2$ (4) $n + 1$

26. For the set of data, 9, 9, 10, 11, 16, which statement is true?
(1) mean > median (2) mean < mode (3) median < mode (4) mean = mode

27. What are the coordinates of the y-intercept for the equation $y + 3x = 6$?
(1) (0,6) (2) (0, −6) (3) (0,3) (4) (0, −3)

28. The value of 6! is
(1) 6 (2) 30 (3) $\frac{1}{6}$ (4) 720

29. The circumference of a circle is 20π. What is the area of the circle?
(1) 10π (2) 20π (3) 100π (4) 400π

30. Given four geometric figures: a square, a rectangle, a trapezoid, and a circle. If one figure is selected at random, what is the probability that the figure has four right angles?
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 0

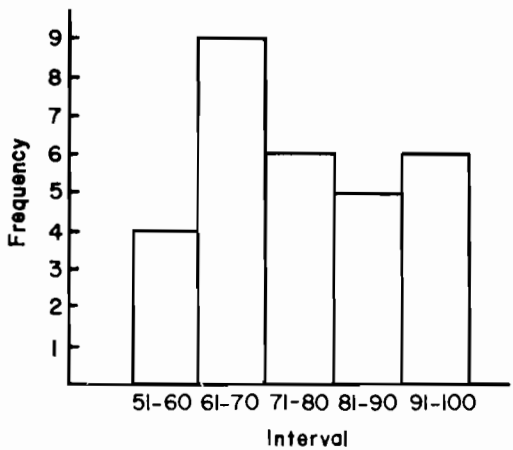
31. Which are factors of $15y^2 - 5y$?
    (1) $5y - 1$ and $3y + 5$ (3) $5y$ and $3y$
    (2) $5y$ and $3y - 1$ (4) $5y - y$ and $3y + 5$
32. A triangle has a base of 12 centimeters and an area of 24 square centimeters. What is the height of the triangle?
    (1) 6 cm (2) 2 cm (3) 3 cm (4) 4 cm
33. Which inequality is equivalent to $2x - 1 > 5$? (1) $x > 6$ (2) $x > 2$ (3) $x < 3$ (4) $x > 3$
34. The sum of $6\sqrt{6}$ and $\sqrt{54}$ equals (1) $3\sqrt{6}$ (2) $6\sqrt{60}$ (3) $9\sqrt{6}$ (4) $15\sqrt{6}$
35. In the truth table below, which is a correct heading for column III?
    (1) $p \rightarrow q$ (2) $p \longleftrightarrow q$ (3) $p \wedge \sim q$ (4) $\sim p \vee q$

Column I	Column II	Column III
p	q	?
T	T	T
T	F	F
F	T	F
F	F	T

Part II

Answer four questions from this part. Show all work unless otherwise directed.

36. Solve the following system of equations graphically and check: $x + y = 1$, $2x - y = 8$ [8,2]
37. Find the three largest consecutive integers whose sum is less than 86. [*Only an algebraic solution will be accepted.*] [5,5]
38. The frequency histogram below shows the distribution of scores on a math test.

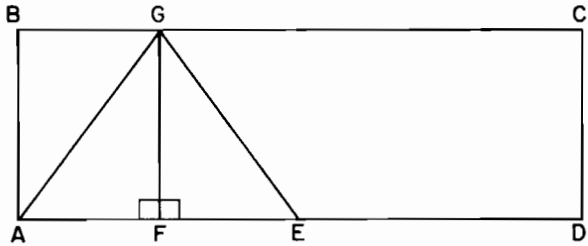


a. *On your answer paper,* copy and complete the table. [2]
b. How many students took the math test? [2]
c. How many students scored above 80? [2]
d. Using the table completed in part *a*, draw a cumulative frequency histogram. [4]

Scores	Frequency	Cumulative Frequency
51-60		
61-70		
71-80		
81-90		
91-100		

39. One positive number is 5 more than another. The sum of their squares is 53. Find both numbers. [*Only an algebraic solution will be accepted.*] [5,5]

40. In the accompanying diagram, $ABCD$ is a rectangle and AGE is an isosceles triangle with $AG = EG$, $\overline{GF} \perp \overline{AD}$, E is the midpoint of $\overline{AD}$, $AF = FE$, $AB = 8$, and $AD = 24$.



a. What is the length of $\overline{AE}$? [2]
b. What is the length of $\overline{AF}$? [2]
c. What is the length of $\overline{AG}$? [2]
d. What is the area of triangle AGE? [2]
e. What is the area of trapezoid $ADCG$? [2]

41. The letters A, E, N, T are written on four individual cards and placed in a container. Each has an equal likelihood of being drawn. One card is drawn from the container, the letter noted, and then the card is returned to the container. A second card is drawn and the letter noted.
a. Make a tree diagram or list the sample space showing all possible outcomes after both drawings. [4]
b. Find the probability that: (1) both letters drawn are the same [2]
(2) the first letter drawn is A and the second letter is T [2]
(3) for both drawings the letter N does not appear [2]

42. *a.* *On your answer paper,* copy and complete the truth table for the statement $[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$. [8]

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$(p \rightarrow q) \wedge \sim p$	$[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$
T	T					
T	F					
F	T					
F	F					

b. Using your results from part *a*, is $[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$ a tautology? [1]
c. Justify the answer you gave in part *b*. [1]