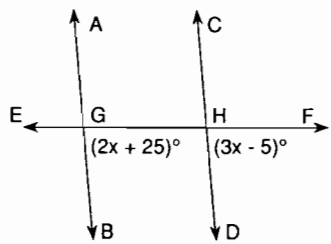
HIGH SCHOOL MATHEMATICS: COURSE I—JANUARY 1991 (1)

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1. In 3 hours a car traveled 180 kilometers. At the same average rate, how many kilometers can the car travel in 5 hours?

2. In the accompanying diagram, parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are intersected by transversal $\overleftrightarrow{EF}$ at points G and H, respectively. If $m\angle FGB = 2x + 25$ and $m\angle FHD = 3x - 5$, find x.

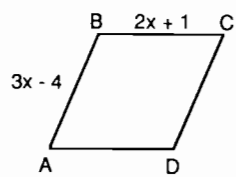


3. Solve for x: $0.03x - 1.2 = 0.24$

4. If $a = -2$ and $b = 3$, what is the value of $-3a^2b$?

5. Solve for m: $\frac{2}{3}m - 4 = 14$

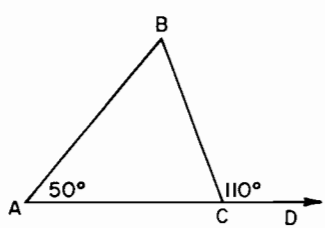
6. In the accompanying diagram of rhombus $ABCD$, the lengths of the sides $\overline{AB}$ and $\overline{BC}$ are represented by $3x - 4$ and $2x + 1$, respectively. Find the value of x.



7. Perform the indicated operations and express as a trinomial:
$$(x + 4)(x - 2) + 3x$$

8. Two numbers are in the ratio 5:6. If the sum of the numbers is 66, find the value of the *larger* number.

9. In the accompanying diagram of $\triangle ABC$, the measure of exterior angle BCD is 110 and $m\angle BAC = 50$. Find $m\angle ABC$.



10. Express $9 - y^2$ as the product of two binomial factors.

11. The measure of the vertex angle of an isosceles triangle is three times the measure of a base angle. Find the number of degrees in the measure of a base angle.

12. Express, in terms of x, the mean of $(4x - 6)$, $(2x + 3)$, and $(3x + 3)$.

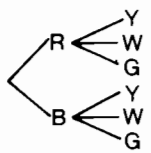
13. The lengths of the sides of a triangle are 3, 4, and 6. If the length of the shortest side of a similar triangle is 5, find the length of its longest side.

14. If $\sqrt{84}$ is simplified to $a\sqrt{b}$ such that a and b are integers, what is the value of a?

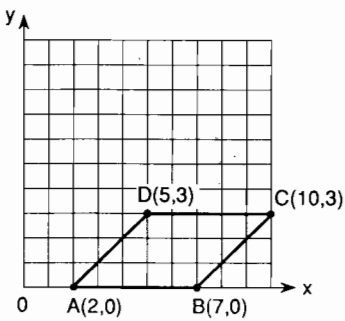
15. Find the numerical value of ${}_4P_3$.

16. For which value of x is the expression $\frac{3}{x + 1}$ undefined?

17. Mary has 2 blouses (1 red and 1 blue) and 3 pairs of slacks (1 yellow, 1 white, and 1 green). The tree diagram at the right represents the outfits she can wear. If Mary chooses 1 blouse and 1 pair of slacks at random, what is the probability that the outfit she chooses will include a pair of green slacks?



18. In the accompanying diagram, $ABCD$ is a parallelogram with vertices $A(2, 0)$, $B(7, 0)$, $C(10, 3)$, and $D(5, 3)$. What is the area of parallelogram $ABCD$?



Directions (19–35): For *each* question chosen, write on your answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

19. The best description of a dilation of a figure is
    (1) an enlargement or a reduction of the figure
    (2) a slide of the figure
    (3) a turning of the figure about some fixed point
    (4) a mirror image of the figure

20. Which property is illustrated by the equation $3x - 6y = 3(x - 2y)$?
    (1) associative (2) commutative (3) distributive (4) closure

21. Which ordered pair is the solution to this system of equations?

$$y = x + 4$$
$$x + y = 2$$

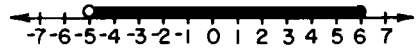
    (1) (1, 5) (2) (0, 2) (3) $(-1, 3)$ (4) $(-4, 0)$

22. Let p represent "x is odd" and let q represent "$x > 15$." Which statement is true if $x = 13$?
    (1) $p \wedge \sim q$ (2) $\sim p \vee q$ (3) $p \rightarrow q$ (4) $p \wedge q$

23. Expressed as a single fraction in lowest terms, the sum of $\frac{3x}{4}$ and $\frac{2x}{3}$ is equivalent to

(1) $\frac{5x}{7}$ (2) $\frac{5x}{12}$ (3) $\frac{17x}{7}$ (4) $\frac{17x}{12}$

24. Which inequality is represented by the graph?



(1) $-5 < x < 6$ (2) $-5 \leq x \leq 6$
(3) $-5 \leq x < 6$ (4) $-5 < x \leq 6$

25. In the following table, which interval contains the median?
(1) 1–5 (2) 6–10 (3) 11–15 (4) 16–20

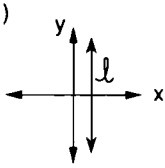
Interval	Frequency
16–20	1
11–15	3
6–10	3
1–5	2

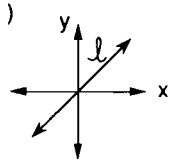
26. The converse of $m \rightarrow \sim r$ is
(1) $m \rightarrow r$ (2) $\sim r \rightarrow m$ (3) $\sim m \rightarrow r$ (4) $r \rightarrow \sim m$

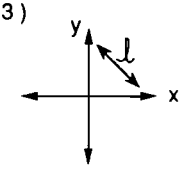
27. The expression $\frac{15k^3 - 9k^2 + 3k}{3k}$, $k \neq 0$, is equivalent to
(1) $5k^2 - 3k + 1$ (2) $5k^2 - 3k$ (3) $15k^3 - 9k^2$ (4) $3k$

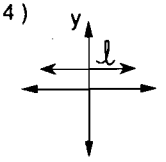
28. If the length of any rectangle is increased by 2 and the width is unchanged, the perimeter is
(1) increased by 2 (2) multiplied by 2 (3) increased by 4 (4) multiplied by 4

29. Which line ℓ has a slope of zero?

(1) 

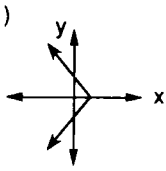
(2) 

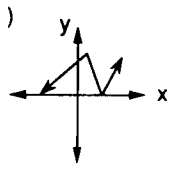
(3) 

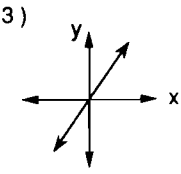
(4) 

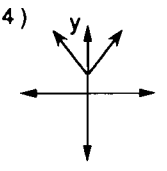
30. The measure of an angle is represented by x. The measure of the complement of this angle can be represented as
(1) $(90 - x)°$ (2) $(x - 90)°$ (3) $(180 - x)°$ (4) $(x - 180)°$

31. Which is a graphic representation of "y varies directly as x"?

(1) 

(2) 

(3) 

(4) 

32. Which statement must *always* be true?
(1) $p \rightarrow q$ (2) $q \wedge \sim q$ (3) $\sim p \rightarrow \sim q$ (4) $p \vee \sim p$

33. The base of a triangle is 4 units more than the height. The area of the triangle is 48 square units. If the height is represented by x, which equation could be used to find the measure of the height of the triangle?
(1) $x(x + 4) = 48$ (2) $\frac{1}{2}x(x + 4) = 48$ (3) $\frac{1}{2}(2x + 4) = 48$ (4) $\frac{1}{2}x(x - 4) = 48$

34. The area of a circle is represented by 16π. What is the length of a diameter of the circle?
(1) 16 (2) 8 (3) $4\sqrt{2}$ (4) 4

35. Which number expresses 72 kilometers per hour as meters per hour?
(1) 7.2×10^{-2} (2) 7.2×10^{2} (3) 7.2×10^{-4} (4) 7.2×10^{4}

Part II

Answer four questions from this part. Show all work unless otherwise directed. [40]

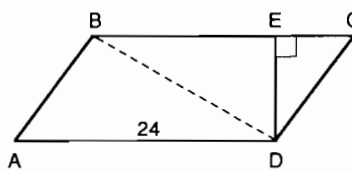
36. *a.* On the same set of coordinate axes, graph the following system of inequalities:

$$y + x \geq 5$$
$$y < 2x + 3 \quad [8]$$

b. Based on the graph drawn in part *a*, write the coordinates of a point in the solution set. [2]

37. If the length of one side of a square is doubled and the length of an adjacent side is decreased by 3, the area of the resulting rectangle exceeds the area of the original square by 16. Find the length of a side of the original square. [*Only an algebraic solution will be accepted.*] [5, 5]

38. In the accompanying diagram, $ABCD$ is a parallelogram, $\overline{DE} \perp \overline{BC}$, diagonal $\overline{BD}$ is drawn, and $AD = 24$.



a. If $AB = 2x + 4$ and $CD = x + 7$, find the length of $\overline{CD}$. [2]
b. If the lengths of $\overline{EC}$ and $\overline{BE}$ are in the ratio 1 : 3, find the length of $\overline{EC}$. [2]
c. Find the length of $\overline{ED}$. [2]
d. Find the area of $\triangle ABD$. [2]
e. Find the area of trapezoid $ABED$. [2]

39. The cost of a telephone call from Wilson, New York, to East Meadow, New York, is \$0.60 for the first three minutes plus \$0.17 for each *additional* minute. What is the greatest number of whole minutes of a telephone call if the cost cannot exceed \$2.50? [*Only an algebraic solution will be accepted.*] [4, 6]

40. *a.* How many different ways can the letters of the word "CHORD" be arranged? [2]
b. How many of the arrangements in part *a* begin with either an "H" or an "O"? [2]
c. If one of the arrangements in part *a* is selected at random, what is the probability it will begin with "C"? [2]
d. If one letter is selected at random from the letters of the word "**CHORD**," find the probability that it will have
(1) horizontal line symmetry, only [1]
(2) vertical line symmetry, only [1]
(3) both horizontal line and vertical line symmetry [1]
(4) neither horizontal line nor vertical line symmetry [1]

41. *a.* *On your answer paper*, construct and complete the truth table for the statement $(\sim p \rightarrow q) \leftrightarrow (p \vee q)$. [9]
b. Based on the truth table constructed in part *a*, is $(\sim p \rightarrow q) \leftrightarrow (p \vee q)$ a tautology? [1]

42. In 20 games, a basketball player scored these points: 36, 32, 28, 30, 33, 36, 24, 33, 29, 30, 30, 25, 34, 36, 34, 31, 36, 29, 30, 34.

a. *On your answer paper*, copy and complete the table to find the frequency in each interval. [2]
b. Construct a frequency histogram using the table completed in part *a*. [4]
c. In what percent of the games played did the player score less than 29 points? [2]
d. Which interval contains the 60th percentile? [2]

Interval	Tally	Frequency
35–37		
32–34		
29–31		
26–28		
23–25		