# HIGH SCHOOL MATHEMATICS: COURSE I—JANUARY 1985 (1)
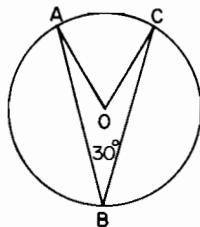
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in terms of π or in radical form.

1. A car rental agency has 5 compact cars, 3 regular size cars, and 2 station wagons for rent. If a car is rented at random, what is the probability it is a compact?

2. In the accompanying diagram, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E, and m$\angle AEC = 24$. If m$\angle DEB = 2x - 6$, find x.
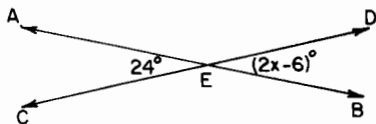

3. If the mean of four scores is 85, what is the total of the four scores?

4. Evaluate: 5!

5. The ratio of Tom's shadow to Emily's shadow is 3 to 2. If Tom is 180 centimeters tall, how many centimeters tall is Emily?

6. If Pat correctly answered 24 out of 30 questions, what percent of the questions did she answer correctly?

7. Express, in cubic feet, the volume of a room whose dimensions are 12 feet long by 10 feet wide by 8 feet high.

8. If $a = 1$ and $b = -2$, find the value of $(a + b)^2$.

9. A single six-sided die is rolled. What is the probability that the outcome is a number less than 12?

10. Solve the following system of equations for x: $x + y = 4$
$$5x - y = 20$$

11. Solve for x in terms of a, b, and c: $ax + b = c$

12. Simplify by combining like terms: $(3a + b) - (a + b)$

13. Two angles of a right triangle are congruent. What is the number of degrees in the measure of each of these angles?

14. Solve for x: $2x - 4 = 4x + 4$ 15. Solve for x: $x + \frac{1}{2} = \frac{10}{4}$

16. A rectangle has an area of $x^2 - 4$. If its length is $x + 2$, express its width in terms of x.

17. Solve for y: $0.02y - 1.5 = 8$ 18. Factor: $x^2 - x - 6$

19. In the accompanying diagram of circle O, m$\angle ABC = 30$. Find the number of degrees in the measure of acute angle AOC.

20. What is the perimeter, in centimeters, of a square whose area is 36 square centimeters?

21. In the accompanying diagram, angle BCD is an exterior angle of triangle ABC. If the measure of angle A is $25°$ and the measure of angle B is $85°$, find the number of degrees in the measure of angle BCD.
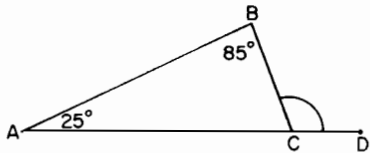
Directions (22-35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

22. Given the true statement: "If John lives in Buffalo, then he lives in New York State."
If John does not live in New York State, which statement is a logical conclusion?
    (1) He does not live in Buffalo.
    (2) He might still live in Buffalo.
    (3) He lives in Chicago.
    (4) He does not live in the United States.

23. The expression "p and q" may be written as
    (1) $p \vee q$ (2) $p \rightarrow q$ (3) $p \leftrightarrow q$ (4) $p \wedge q$
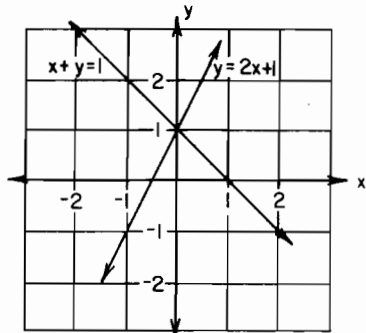
24. What is the mode for the following data?

    19, 8, 38, 41, 8, 16, 3

    (1) 7   (2) 8   (3) 16   (4) 19

25. Which expression is equivalent to $6x + 6y$?
    (1) $12xy$ (2) $6(x + y)$ (3) $6xy$ (4) $6x + y$

26. The product of $5x^2$ and $3x^5$ is
    (1) $8x^7$ (2) $8x^{10}$ (3) $15x^7$ (4) $15x^{10}$

27. Which is an equation of a line whose slope is -2?
    (1) $y = 2x - 1$ (2) $y = -2x + 1$ (3) $y = x - 2$ (4) $y - 2x = 4$

28. As shown on the accompanying graph, what is the solution of the system of equations $y = 2x + 1$ and $x + y = 1$?
    (1) $(1,0)$ (2) $(-1,0)$ (3) $(0,1)$
    (4) $(0,-1)$
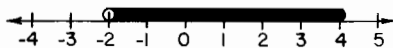

29. The coordinates of the vertices of a triangle are $(1,1)$, $(3,1)$, and $(3,5)$. The triangle formed is
    (1) a right triangle   (3) an isosceles triangle
    (2) an obtuse triangle   (4) an equilateral triangle

30. Which is undefined when $x = 2$?
    (1) $x - 2$ (2) $\dfrac{1}{x - 2}$ (3) $\dfrac{x}{2}$ (4) x^0

31. Which inequality is the solution set of the graph shown below?
    (1) $-2 < x < 4$ (2) $-2 \leq x < 4$ (3) $-2 \leq x \leq 4$ (4) $-2 < x \leq 4$

32. The length of the hypotenuse of a right triangle is 7 and the length of one leg is 4. What is the length of the other leg?
    (1) 11   (2) $\sqrt{65}$ (3) 3   (4) $\sqrt{33}$

33. The solution set of $x^2 - 3x + 2 = 0$ is
    (1) {2,1}   (2) {$-2, -1$}   (3) {$-2, 1$}   (4) {2, -1}

34. Which of the following is true when p is false and q is false?
    (1) $p \vee q$ (2) $p \wedge q$ (3) $p \leftrightarrow q$ (4) $\sim (\sim p)$

35. The expression $\sqrt{150}$ is equivalent to
    (1) $5\sqrt{6}$ (2) $15\sqrt{10}$ (3) $25\sqrt{6}$ (4) 75

Part II

Answer four questions from this part. Show all work unless otherwise directed.

36. Solve the following system of equations graphically and check:

$$x + y = 12$$
$$y = 3x \qquad [8, 2]$$

37. One integer is 3 more than twice another integer. The sum of these integers is greater than 24. Find the *smallest* values for these integers. [*Only an algebraic solution will be accepted.*]   [5, 5]

38. In the diagram, $\overline{PR}$ is a diameter of circle O. The measure of $\angle QOR$ is $(4x - 10)°$ and the measure of $\angle QOP$ is $(x + 50)°$.
    a. What is the value of x?   [3]
    b. How many degrees are in the measure of minor arc QP? [2]
    c. How many degrees are in the measure of $\angle QRP$?   [2]
    d. If the area of circle O is 16π, find the circumference in terms of π.   [3]

39. In triangle ABC, the ratio of the measure of angle A to the measure of angle B is 3:5. The measure of angle C is 20 more than the sum of the measures of angles A and B. What is the measure of each angle in triangle ABC? [*Only an algebraic solution will be accepted.*]   [5, 5]

40. A softball team plays two games each weekend, one on Saturday and the other on Sunday. The probability of winning on Saturday is $\frac{3}{5}$ and the probability of winning on Sunday is $\frac{4}{7}$.

    What is the probability of:
    a. losing a Saturday game   [2]
    b. losing a Saturday game and winning a Sunday game   [2]
    c. winning a Sunday game after already winning a Saturday game   [2]
    d. winning both games   [2]
    e. losing both games   [2]

41. The following table represents the ages of students hired for various summer jobs at a State park in New York:

Age Interval	Cumulative Frequency
11–12	2
13–14	6
15–16	9
17–18	15
19–21	20

   a. On your answer paper, draw a cumulative frequency histogram.   [4]
   b. What was the total number of students hired?   [2]
   c. In which interval does the median lie?   [2]
   d. How many students age 13 or 14 were hired?   [2]

42. *a. On your answer paper*, copy and complete the truth table for the statement
   $\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$.   [8]

p	q	$\sim q$	$p \rightarrow q$	$\sim (p \rightarrow q)$	$p \wedge \sim q$	$\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$
T	T					
T	F					
F	T					
F	F					

   b. What is logically equivalent to the negation of:
      "If I receive a scholarship, then I will go to college"?   [2]
     (1) If I do not receive a scholarship, then I will not go to college.
     (2) I receive a scholarship and I will not go to college.
     (3) It is not the case that I receive a scholarship and I go to college.
     (4) If I do not go to college, then I did not receive a scholarship.